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CRACK MODELLING BY HYBRID-TREFFTZ STRESS FINITE ELEMENTS

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In plane elasticity, assuming negligible body forces and small strains, stresses derived from any biharmonic function verify compatibility and equilibrium conditions, as well an elastic constitutive law [1]. To numerically approximate the stress distribution using hybrid-Trefftz stress finite elements [2, 3, 4], we combine regular basis functions derived from the biharmonic Airy function. To model local effects, typically high stress gradients associated with fracture processes, particular solutions can be derived from a set of biharmonic functions [5] and used to enrich the finite element approximation basis and speed up convergence. In particular, the Williams solution of a semi-infinite crack in [6] is used to model small imperfections in plates, typically appearing in fracture benchmark tests to induce crack propagation. The corresponding case with an elastic filler is discussed in [7], and we use the extended solution presented therein to model crack repair. Further solutions can be determined by the complex representation of stresses [8] together with the Westergaard approach of a single complex stress function [9]. Of particular importance are Westergaard local solutions given in [10] to model embedded crack behaviour, also known as the classical Griffith problem [11].

Besides the stress field, boundary displacements are also independently approximated in hybrid stress formulations. Simple polynomial functions can be used for this purpose such as Tchebychev polynomials. However, we use Bernstein polynomials near stress concentrations on the crack tips, as they better fit the corresponding boundary displacements when the numerical method converges. Further details on the formulation can be found in [12].

We present developments on filled cracks, complex solutions for the Griffith problem, and fracture propagation. Figure 1 represents an application of the method to a plate with an edge-crack assuming K-field dominance [13]. As pointed out in [12], the stress intensity factors [14] are computed directly from the stress approximation weights. For the open crack case, the numerical stress intensity factor in mode I fracture is $K_{\mathrm{I}} = 0.6648$, very accurate when compared with $K_{\mathrm{I}} = 0.6646$ in [15], dropping to $K_{\mathrm{I}} = 0.4130$ for the case of a crack filled with an elastically weak material. The method recovers $K_{\mathrm{II}} = 0$ in both cases.

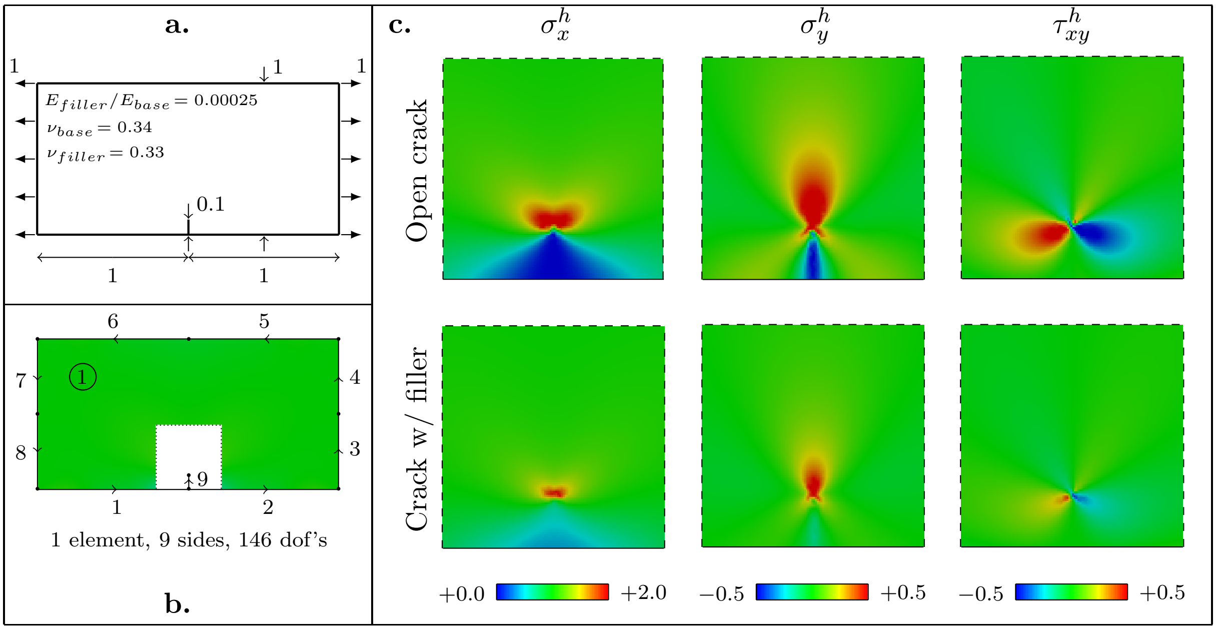

Figure 1: Plane stress specimen with an edge-crack. **a.** Geometry, external loading, Young's modulus E and Poisson's ratio ν. **b.** Discretization. **c.** Stress fields next to the crack tip.

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