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OPTIMAL PROJECTIONS FOR REDUCED ORDER MODELS

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A typical strategy for developing reduced-order models (ROMs) for a multi-parameter system of partial differential equations involves the following steps [1]: (a) the generation of several snapshots corresponding to the solution of the original system at specified values of the parameters, (b) construction of a basis for the ROM from these snapshots (using proper orthogonal decomposition, for example), (c) projection of the original system on to this finite-dimensional basis, and (d) selection of a new value for the set of parameters for the generation of new snapshot(s)/basis functions in order to improve the accuracy of the ROM.

In this talk we consider the choice of the "optimal" projection in the step (c) above. In particular we consider the following question: *Given a certain basis, and a criterion for the optimality for the ROM solution, what is the form of the projection that will guarantee this optimal ROM solution?* We obtain a definition of this projection by relying on similar ideas that have been developed in the context of the variational multiscale (VMS) method [2]. The crucial difference is that in the VMS method the projection is from an infinite-dimensional function space to a finite dimensional subspace, whereas in the proposed ROM, the projection is from a large finite dimensional function space to a much smaller finite dimensional subspace.

After deriving the optimal projection, we note:

1. It differs from the Galerkin projection owing to the presence of terms that are driven by the scales not represented by the ROM basis.
2. These additional terms involve the inverse of the discrete form of the PDE posed in the original large finite dimensional function space. The explicit calculation of this

inverse is clearly not practical. However it may be replaced by simple approximations that yield projections that are of the same computational complexity as the Galerkin projection.

3. This approach also yields an approximation to the error in the ROM solution which can be used to select the next value for the set of parameters in order to obtain a new snapshot (step (d) above).

We demonstrate the utility of these ideas by developing ROMs for PDE systems with multiple parameters.

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