

# A PENETRATION-FREE NONSMOOTH DYNAMICS METHOD FOR FRICTIONLESS CONTACT/IMPACT PROBLEMS

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**Key words:** *Nonsmooth Contact Dynamics, Flexible Multibody Systems, Time Integration, Generalized- $\alpha$  Method, Time-Stepping Schemes, Index Reduction.*

This paper studies numerical algorithms for the simulation of mechanical systems including rigid and flexible bodies, kinematic joints and frictionless contact conditions. The condition of impenetrability of the bodies in contact is expressed as a unilateral constraint, with the consequence that impacts and/or instantaneous changes in the velocities may arise in the dynamic response.

When dynamic contacts are analyzed between elastic solids and structures, the gap velocity is necessarily discontinuous otherwise a non-physical penetration of the bodies would occur. When contacts between rigid bodies are studied, e.g., in multibody systems, impulsive reaction forces can also occur leading to an instantaneous change in the linear and angular momenta of each body. Standard schemes, such as the Newmark, HHT or generalized- $\alpha$  methods, are not consistent in these cases since the numerical response may artificially generate energy when a contact occurs. In order to analyze impact phenomena, nonsmooth time integration methods have been proposed in the literature and can be classified into two main groups, namely, event-driven schemes and time-stepping schemes. Event-driven schemes are accurate for the free flight smooth motions and are especially suitable for small multi-body systems with a limited number of events, but they become inefficient if frequent transitions occur in a short time. Time-stepping methods, such as Moreau–Jean scheme, have been proven to be convergent and robust even for a large number of events, and are extensively applied as the solution to nonsmooth system models.

A fundamental property of the Moreau–Jean scheme is that the unilateral constraints are

imposed at velocity level. The consequence is that some penetration can be observed in the numerical solution, which may not be physically acceptable. In order to prevent such penetration problems, this paper presents an algorithm which enforces the constraint not only at velocity level, so as to inherit good consistency and stability properties, but also at position level. For that purpose, the Gear-Gupta-Leimkuhler (GGL) approach, which was initially developed for the stabilization of index-3 differential-algebraic equations, is generalized to systems with unilateral constraints following a similar idea as in [1, 3]. However, the present work differs from [1, 3] by the formulation of specific complementarity conditions at position and velocity levels, as well as by the construction of an implicit algorithm which solves the dynamic equilibrium and the complementarity conditions in a monolithic way using a semi-smooth Newton process at each time step.

In order to improve the accuracy in the smooth part of motion, we also follow the strategy proposed by Chen et al [2], which involves a partitioning of the generalized forces in the dynamic equilibrium into smooth terms and nonsmooth (impulsive) terms. The generalized- $\alpha$  method is used to integrate the smooth terms whereas all impulsive terms are treated using an Euler implicit integration scheme to ensure consistency. This means that different integration formulae are used for the different contributions to the equation of motion. Nevertheless, the advantage of this approach is that the numerical dissipation of the generalized- $\alpha$  method is significantly smaller than the numerical dissipation of the Euler implicit scheme, so that the energy behavior is strongly improved compared to the Moreau-Jean scheme, especially for mechanical systems exhibiting both impacts and structural vibrations. In some sense, in this scheme, the high numerical dissipation of the Euler implicit scheme is only acting locally at the contact region and during the contact time, and not on the full system during the whole trajectory.

The numerical behaviour of the proposed method is studied and compared to other approaches for a number of numerical examples. It is shown that the formulation offers a unified and valid approach for the description of dynamic contact conditions between rigid bodies as well as between flexible bodies.

## REFERENCES

- [1] V. Acary. Projected event-capturing time-stepping schemes for nonsmooth mechanical systems with unilateral contact and Coulomb's friction. *Computer Methods in Applied Mechanics and Engineering*, Vol. **256**, 224–250, 2013.
- [2] Q.-z. Chen, V. Acary, G. Virlez, and O. Brüls. A nonsmooth generalized- $\alpha$  scheme for flexible multibody systems with unilateral constraints. *International Journal for Numerical Methods in Engineering*, Vol. **96**, 487–511, 2013.
- [3] S. Schoeder, H. Ulbrich, and T. Schindler. Discussion on the Gear-Gupta-Leimkuhler method for impacting mechanical systems. *Multibody Systems Dynamics*, Vol. **31**, 477–495, 2014.