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A MODIFIED PERTURBED LAGRANGIAN FORMULATION FOR CONTACT PROBLEMS

M. Tur*, J. Albelda, J.J. Ródenas and J.M. Navarro-Jiménez

Centro de Investiación en Tecnología de Vehículos (CITV)
Universitat Politècnica de València, Camino de Vera, s/n, E-46022, Valencia, Spain
e-mail: matuva@mcm.upv.es, jalbelda@mcm.upv.es, jjrodena@mcm.upv.es

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In general terms, there are two strategies to obtain stable and convergent discretizations of contact problems. The first approach is to correctly choose the discretized space for the displacements and the multiplier in order to fulfill the discrete inf-sup condition with a constant independent of the mesh size. This is achieved by the so-called mortar method that in recent years has been applied to solve frictional and non-frictional contact problems for small and large deformations. On the other hand, if one wish to freely choose the discretized spaces of multipliers and displacements, then the Lagrangian functional of the problem can be modified adding new terms. This is the basis of the stabilization methods [1, 2, 3].

In 1985 Simo, Wriggers and Taylor [4] proposed a perturbed Lagrangian formulation for the solution of contact problems. That formulation can be classified as a stabilized method, preserving the stability of the discretized problem if the penalty parameter, ϵ, is small enough. However, the method has a consistency error that increases if the penalty ϵ becomes smaller.

In this work we propose a modification of the perturbed Lagrangian formulation. The Lagrangian functional of the contact problem is defined as

$$\Pi(\mathbf{u}^h, \lambda^h) = \Pi_p(\mathbf{u}^h) + \int_{\Gamma_c} \lambda\, g(\mathbf{u}^h)\, d\Gamma - \frac{1}{2\epsilon} \int_{\Gamma_c} \left(\lambda^h - T^H\right)\, d\Gamma$$

where $\mathbf{u}$ is the discretized displacement field, λ is the Lagrange multiplier field, $g(\mathbf{u})$ is the gap, $\Pi_p(\mathbf{u})$ is the total potential energy, Γ_c is the contact area and ϵ a penalty parameter. As shown in the above equation, if one chooses T^H as the traction computed from the displacement, the method is the Barbosa-Hughes stabilization [5], that can be considered as a version of the Nitsche method used to solve contact problems in [1].

In this work we propose T^H to be computed as the traction computed from the solution of the previous coarser mesh. This idea was used in [6] to apply Dirichlet boundary conditions in immersed boundary problems. The stability of the method can be demonstrated for small enough values of the penalty parameter ϵ. The convergence of the proposed method is obtained if the correction traction T^H converges to the exact traction of the problem. In practice, this is achieved for a wide range of values of the penalty parameter.

The proposed formulation is easy to implement because the additional term T^H does not depend on the degrees of freedom of the mesh and thus no variations nor linearization of this term is needed to solve the problem. Furthermore, as a stabilized method there are no constraints to choose the Lagrange multiplier field other that having good approximation properties. In this work, we choose a discretization that allows to easily condense the Lagrange multipliers during the assembly of each element leading to a system of equations without the degrees of freedom of the multipliers.

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