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# RBF-BASED MESHLESS APPROACHES FOR FREQUENCY-DOMAIN ANALYSIS OF HEAT CONDUCTION PROBLEMS

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A variety of numerical methods can be applied to study heat conduction in solid media. Some usual approaches apply the FEM or the BEM for that purpose, although several applications of meshfree methods can be found in the literature, including time-domain and transformed-domain formulations. Here, the application of meshless formulations in the frequency domain to address transient heat conduction is discussed, considering the case of both homogeneous media and media with varying properties (such as in FGMs), including the possible presence of cracks, holes or other discontinuities.

One should note that in the presence of spatially variable thermal conductivity values, such as in FGMs, the time-domain governing equation can be written, for the 2D case, as:

$$\nabla \cdot \left( k(\mathbf{x}) \nabla T(\mathbf{x},t) \right) = \rho c(\mathbf{x}) \frac{\partial T(\mathbf{x},t)}{\partial t} , \qquad (1)$$

where $T(\mathbf{x},t)$ is the temperature at domain point $\mathbf{x}$, $k$, $c$ and $\rho$ are the thermal conductivity, specific heat and density, respectively.

Application of a Fourier transform to this equation, moving the problema to the frequency domain, allows establishing the following frequency-dependent equation:

$$\nabla \cdot \left( k(\mathbf{x}) \nabla \hat{T}(\mathbf{x},\omega) \right) - \mathrm{i} \rho c(\mathbf{x}) \omega \hat{T}(\mathbf{x},\omega) = -\rho c(\mathbf{x}) T_0(\mathbf{x}) , \qquad (2)$$

where $\hat{T}(\mathbf{x},\omega)$ is the frequency transformed temperatura, and $T_0$ represents the temperature distribution at instant 0 (initial conditions).

Both strong and weak formulations are here addressed, using meshless methods in which the interpolation is performed by means of RBFs. Local and global interpolation methods are applied, and their performance is analysed. Specifically, Kansa's collocation method [1] and the MLPG [2] (collocation and Heaviside weighted) are applied to study this class of problems, and their results are compared with those computed by a FEM model.

In general, it was observed that a very good convergence is observed for the meshless approaches, usually surpassing the convergence rates observed for the FEM, as can be seen in the sample results shown in Figure 1.

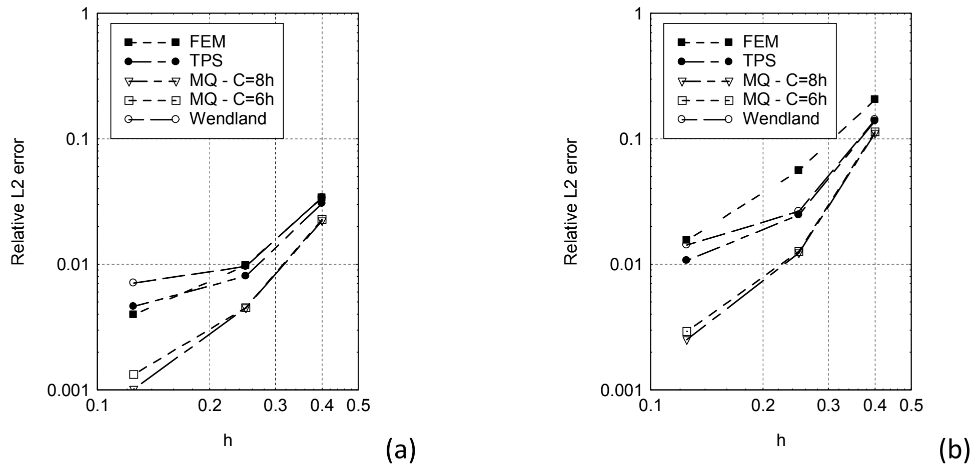


**Figure 1** – convergence of the Heaviside weighted MLPG in a unit circle with an internal source, considering null temperature (a) or null flux (b) boundary conditions.

For the case of non-homogenous media, in which the properties can be position-dependent (as in FGMs), it was also found that the meshless approaches can provide very good results, that perfectly match those computed using standard time-marching algorithms.

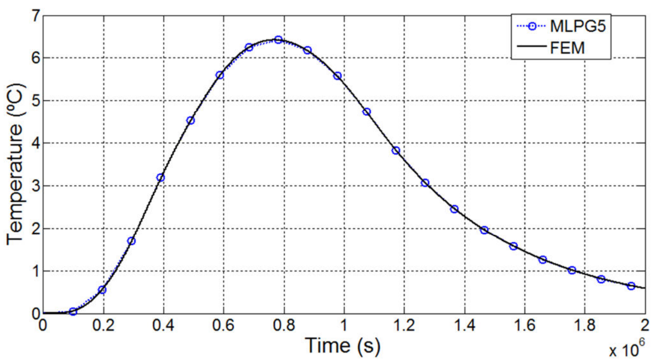


**Figure 2** – Result computed at an internal point of a rectangular FGM using the FEM and MLPG5.

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