11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

IMPLEMENTATION AND VALIDATION OF HIGH-ACCURACY AEROACOUSTIC SCHEMES FOR THE DESCRIPTION OF VISCOUS GASFLOWS

Anatol V. Alexandrov[1] and Ludwig W. Dorodnicyn[2*]

[1]M.V. Keldysh Institute for Applied Mathematics
Miusskaia sq. 4, 125047 Moscow, Russia, alexandrov@imamod.ru
[2]M.V. Lomonosov Moscow State University, Faculty CMC
Vorobievy gory, 119991 Moscow, Russia, dorodn@cs.msu.su

Key words: *Dispersion-Relation-Preserving Schemes, Spectral Resolution, Navier-Stokes Equations, Numerical Boundary Conditions.*

The work is devoted to further development of DRP scheme by C.Tam and coauthors [1] as well as schemes by C. Bogey and C. Bailly [2] aimed at the implementation of the schemes mentioned to nonlinear viscous flows, so far as viscosity plays an important role in sound generation. We will carry out accuracy and stability testing of the numerical algorithms and present some advances in discrete boundary conditions. We concentrate at essentially viscous flows including the 2D Lamb–Oseen and the 3D Taylor–Green vortex decay as well as channel flows.

Following [3], we express the 2D nonlinear Navier–Stokes equations in the form of first derivatives

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \frac{\partial \mathbf{R}}{\partial x} + \frac{\partial \mathbf{S}}{\partial y}\,, \tag{1}$$

where $\mathbf{Q}$ is the vector of conservative variables, $\mathbf{F}$ and $\mathbf{G}$ are the convective fluxes, $\mathbf{R}$ and $\mathbf{S}$ are the viscous fluxes:

$$\mathbf{Q} = \begin{pmatrix} \rho & \rho u & \rho v & E \end{pmatrix}^T,$$

$$\mathbf{F} = \begin{pmatrix} \rho u & \rho u^2 + p & \rho uv & (E{+}p)\,u \end{pmatrix}^T, \qquad \mathbf{G} = \begin{pmatrix} \rho v & \rho uv & \rho v^2 + p & (E{+}p)\,v \end{pmatrix}^T,$$

$$\mathbf{R} = \begin{pmatrix} 0 & \tau_{xx} & \tau_{xy} & u\tau_{xx} + v\tau_{xy} - q_x \end{pmatrix}^T, \qquad \mathbf{S} = \begin{pmatrix} 0 & \tau_{xy} & \tau_{yy} & u\tau_{xy} + v\tau_{yy} - q_y \end{pmatrix}^T.$$

The vectors $\mathbf{R}$ and $\mathbf{S}$ contain the components of viscous stress tensor and heat flux

$$\tau_{xx} = \frac{4}{3}\,\mu\,\frac{\partial u}{\partial x} - \frac{2}{3}\,\mu\,\frac{\partial v}{\partial y}\,, \quad \tau_{xy} = \mu\,\frac{\partial v}{\partial x} + \mu\,\frac{\partial u}{\partial y}\,, \quad \tau_{yy} = -\frac{2}{3}\,\mu\,\frac{\partial u}{\partial x} + \frac{4}{3}\,\mu\,\frac{\partial v}{\partial y}\,,$$

$$q_x = -\lambda\frac{\partial T}{\partial x}\,, \quad q_y = -\lambda\frac{\partial T}{\partial y}\,. \tag{2}$$

High-accuracy schemes with spectral resolution on uniform grids frequently referred to as Dispersion-relation-preserving (DRP) schemes [1] are based on the approximation of first spatial derivatives by wide-stencil centered differences as written below:

$$\left(\frac{\partial u}{\partial x}\right)_j \sim \frac{1}{\Delta x}\sum_{l=1}^{m} a_l\Big(u_{j+l} - u_{j-l}\Big). \tag{3}$$

Here the coefficients a_l are obtained from some spectral-optimization procedure matching the requirement of 4th-order accuracy. Preferably we consider the seven-point ($m = 3$) scheme from [1]. In Eqs. (1)–(2) operators of type (3) replace each x- and y-derivative.

Wide stencils complicate the imposition of discrete boundary conditions. The technique of conditions at artificial boundaries is based on so-called consistent boundary conditions for hyperbolic systems from [4]. Wall boundary conditions for viscous gas develop the ghost-point approach from [3] or, alternatively, use a kind of SBP method.

Schemes with centered differences are known to produce sawtooth oscillatory modes. The latter are not being suppressed by viscosity when the discretization stated above is used. Important whether initial and boundary conditions generate or release such undesired oscillations.

We carry out qualitative and quantitative assessment of the high-accuracy schemes on the known flows of viscous fluid at low Mach numbers and moderate Reynolds numbers. This includes the Lamb–Oseen vortex, the 3D Taylor–Green vortex, and the Poiseuille flow. Such benchmarks can demonstrate expected behavior of the algorithms in their future turbulent jet applications.

REFERENCES

[1] C.K.W. Tam, J.C. Webb. Dispersion-relation-preserving finite difference schemes for computational acoustics. *J. Comput. Phys.*, Vol. **107**, 262–281, 1993.

[2] C. Bogey, C. Bailly. A family of low dispersive and low dissipative explicit schemes for flow and noise computations. *J. Comput. Phys.*, Vol. **194**, 194–214, 2004.

[3] C.K.W. Tam, Zh. Dong. Wall boundary conditions for high-order finite-difference schemes in computational aeroacoustics. *Theor. Comput. Fluid Dynamics*, Vol. **6**, 303–322, 1994.

[4] L.W. Dorodnicyn. Artificial boundary conditions for high-accuracy aeroacoustic algorithms. *SIAM J. Scientific Computing*, Vol. **32**, 1950–1979, 2010.