11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A SINGLE-SYNCHRONIZED LINEAR SOLVER FOR THE SOLUTION OF PROBLEMS OF COMPUTATIONAL MECHANICS ON PARALLEL COMPUTERS

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Key words: *Synchronization, Linear Solver, Parallel Computation, Iteraive method.*

We consider iterative methods for solving a linear system of equations $A\boldsymbol{x} = \boldsymbol{b}$ where $A \in R^{N\times N}$ is a given non-symmetric matrix. Vectors $\boldsymbol{x}$ and $\boldsymbol{b}$ are a solution vector and a right-hand side vector, respectively. These large scale problems often appear in the field of computational mechanics, and we intend to solve efficiently these linear systems on parallel computers. Among many iterative methods, product-type of iterative methods, e.g., BiCGStab and GPBiCG are often used for the purpose of solution for realistic problems. However, number of synchronization per one iteration needs three times. BiCGSafe (with safety convergence) method [1] using the strategy of associate residual was proposed in 2005. In general, BiCGSafe method works well compared with the original GPBiCG method due to adoption of smoothed convergence behavior. Number of synchronization need two times per one iteration. The variants of GPBiCG method improved GPBiCG itself by using three-term recurrence similar to the one for the Lanczos polynomials. These variants of GPBiCG method needs two times synchronization per one iteration. Moreover its variants proposed in 2011 combined the original one with a slightly modified version of the coupled two-term recurrences. One of the variants of GPBiCG method applied the coupled two-term recurrences embodied by Rutishauser [2] in 1959.

In this article, we adopt the above formular for computation of parameters α_k and β_k for the purpose of improvement of performance on parallel computer with distributed memory. Namely we will reduce number of synchronization of BiCGSafe method from two times to single time per one iteration. This strategy becomes effective when number of processors become larger. Our final target is to solve efficiently linear systems which stem from computational mechanics on parallel computers.

We present results of numerical experiments. All computations were done in double precision floating point arithmetic of Fortran90, and performed on Fujitsu PRIMERGY CX400(CPU: Intel Xeon E5-2680, memory: 128Gbytes, OS: Red Hat Linux Enterprise, total nodes: 1476 nodes, cores: 16cores/1node). Fujitsu compiler optimum option "-Kfast" were used. Process parallelization was done by MPI library. Stopping criterion of iterative methods is less than 10^{-8} of the relative residual 2-norm $||\boldsymbol{r}_{k+1}||_2/||\boldsymbol{b} - A\boldsymbol{x}_0||_2$. In all cases the iteration was started with the initial guess solution $\boldsymbol{x}_0 = (0, 0, \ldots, 0)^T$. Measurement of the elapsed time was done by system function of gettimeofday. All test matrices were normalized with diagonal scaling. Number of process varied as 1, 16, 32, 64 and 128. Measurements of the elapsed time per each matrix were five times. TRR (True Relative Residual) for the approximated solutions $\boldsymbol{x}_{k+1}$ means $\log_{10}(\|\boldsymbol{b} - A\boldsymbol{x}_{k+1}\|/\|\boldsymbol{b} - A\boldsymbol{x}_0\|)$. We see that the original BiCGSafe method and a single-synchronized BiCGSafe method outperform compared with the conventional GPBiCG and its variant methods in view of parallel efficiency and accuracy of the approximated solutions.

Table 1: Parallel performance of some iterative methods for matrix: tmt_unsym with dimension of 917,825.

method	np	Mv	total time [sec.]	ratio	ave.time [msec.]	speed-up	$\log_{10}$ (TRR)
GPBiCG	1	8,900	210.014	1.00	23.60	1.00	-6.73
	16	10,160	30.072	1.00	2.96	7.97	-6.31
	32	9,062	11.827	1.00	1.31	18.08	-5.28
	64	9,238	**4.413**	1.00	0.48	49.40	-5.89
	128	9,076	2.481	1.00	0.27	86.32	-5.79
GPBiCG_variant	1	10,182	272.586	1.30	26.77	1.00	-7.55
	16	9,842	28.885	0.96	2.93	9.12	-7.50
	32	9,044	11.893	1.01	1.32	20.36	-7.56
	64	8,940	4.870	1.10	0.54	49.14	-7.56
	128	8,982	2.905	1.17	0.32	82.77	-7.17
BiCGSafe	1	10,054	220.194	1.05	21.90	1.00	-7.55
	16	9,056	**23.021**	0.77	2.54	8.62	-7.60
	32	8,848	**10.183**	0.86	1.15	19.03	-7.49
	64	11,140	4.911	1.11	0.44	49.68	-6.93
	128	9,210	**2.252**	0.91	0.24	89.57	-7.54
BiCGSafe-1sync	1	8,718	**194.084**	0.92	22.26	1.00	-7.18
	16	8,942	24.815	0.83	2.78	7.82	-7.58
	32	8,764	11.171	0.94	1.27	17.37	-7.35
	64	9,378	**4.441**	1.01	0.47	43.70	-7.59
	128	9,084	2.289	0.92	0.25	84.79	-7.18

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