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3D *P*-ADAPTION FOR COMPRESSIBLE FLOW

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We present a p-adaptive method which takes advantage of the ability of a discontinuity sensor used to quantify the difference between the actual solution (p) and the projected reduced one ($p-1$) in order to vary the polynomial resolution in an element. The value of the sensor in an element is defined as [1]:

$$S_e = \frac{||\rho_e^p - \rho_e^{p-1}||_{L_2}}{||\rho_e^p||_{L_2}} \tag{1}$$

where ρ_e^p and ρ_e^{p-1} are the average solutions of degree p and $p-1$ respectively on the same element. The polynomial degree is decreased when a discontinuity is present in order to avoid oscillations and increased when a high gradient is identified to improve the accuracy. This procedure allows the simulation to adapt to the flowfield, increasing the accuracy of the solution only where needed and, as a consequence, reducing the computational cost required for solving the problem.

Initially, a converged, low order, solution is obtained after which the sensor in each element is calculated. Based on the determined sensor value and the pre-defined sensor thresholds, the degree of the polynomial approximation in each element is increased, reduced or maintained and a new converged solution is obtained. where s_{ds}, s_{sm} and s_{fl} are the threshold values to identify discontinuities, smooth and flat solutions respectively. This procedure is carried out iteratively.

The solution at $p-1$ is acquired by projecting the solution at polynomial order p, determined using a modified basis, onto an orthogonal expansion basis [2]. Hence, using conservation arguments, the solution of a general variable u is expressed as $u^0 = u$ where u^0 and u represent the general solution obtained using a orthogonal and modified basis respectively, hence:

$$\mathbf{B}^0\hat{u}^0 = \mathbf{B}\hat{u} \rightarrow \hat{u}^0 = \left[\mathbf{B}^0\right]^{-1}\mathbf{B}\hat{u} \tag{2}$$

where $\hat{u}$ represents the vector of coefficients. Since the coefficients are not coupled in the orthogonal basis and the information about the mean is contained only in the first coefficient, it is possible to apply a filter to the orthogonal coefficient vector. The cut-off filter sets all the coefficients that are higher that $p-1$ equal to zero. The orthogonal coefficients represent the solution at $p-1$ using an orthogonal basis, hence the following transformation has to be applied to obtain the coefficients that belong to the modified basis:

$$\hat{u}_f = \left[\mathbf{B}^{-1}\right]\mathbf{B}^0 \hat{u}_f^0 \tag{3}$$

where $\hat{u}_f$ represents the filtered coefficient that belong to the modified basis. In this way, the information contained in the high frequency components is removed without altering the mean value. this is particularly required when computing the advective numerical fluxes on the interface of two elements with different expansions since the appropriate number of quadrature points has to be used to avoid numerical instabilities [3].

The performance of the p-adaptive method is tested and illustrated using subsonic flow past an ONERA M6 wing geometry. This illustrates that the algorithm is working for hybrid meshes consisting of prism and tetrahedral shaped elements. Furthermore, the performance in parallel is tested and currently the time-dependent algorithm is considered to be work in progress.

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