11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

GLOBAL ENERGY MINIMIZATION FOR MULTI-CRACK GROWTH IN LINEAR ELASTIC FRACTURE USING THE EXTENDED FINITE ELEMENT METHOD

Danas Sutula*[1], Pierre Kerfriden[2] and Stephane P.A. Bordas[3]

[1] Cardiff University, The Parade, CF24 3AA, United Kingdom, sutulad@cardiff.ac.uk
[2] Cardiff University, The Parade, CF24 3AA, United Kingdom, kerfridenp@cardiff.ac.uk
[3] University of Luxembourg, Research Unit in Engineering, stephane.bordas@uni.lu

Key words: *crack propagation, interacting cracks, energy release rate, extended finite element method, linear elasticity, energy minimisation.*

In computational fracture mechanics as applied, for example, to damage tolerance assessment, it has been common practice to determine the onset of fracture growth and the growth direction by post-processing the solution of the linear elastostatics problem, at a particular instance in time. For mixed mode loading the available analytically derived criteria that can be used for determining the onset of crack growth typically rely on the assumptions of an idealized geometry e.g. a single crack subjected to remote loading [1, 2] and that the kink angle of the infinitesimal crack increment is quite small [3]. Moreover, the growth direction given by a criterion that is based on an instantaneous local crack tip field can only be valid for infinitesimally small crack growth increments. Consequently, the maximum hoop stress criterion [4] and other similar criteria [5] disregard the changes in the solution that take place as fractures advance over a finite size propagation. Hence, due to the error committed in time-integration, fractures may no longer follow the most energetically favorable paths that theoretically could be achieved for a specific discrete problem.

In our approach, we investigate multiple fracture evolution under quasi-static conditions in an isotropic linear elastic solid based on the principle of minimum potential elastic energy, which can help circumvent the aforementioned difficulties. The technique enables a minimization of the potential energy with respect to all crack increment directions minding their relative interactions. The directions are optimized (in the energy sense) by considering virtual crack rotations to find the energy release rates and its first derivatives in order to determine, via an iterative process, the directions that yield zero energy release rates with respect to all virtual rotations [6]. We use the extended finite element method (XFEM) [7, 8] for discretization of a 2D continua in order to model an elaborate crack evolution over time, similar in principle to [9], although we consider here hundreds of propagating cracks. Although XFEM facilitates mesh independent fracture propagation the enrichment must be updated at each time step. In the current implementation this is achieved by means of a systematic book-keeping of the element enrichment data, addition and removal of enrichment only where necessary, and a consistent updating of the global system of equations. Consequently, moderate computational times are obtained, even in our *Matlab* implementation. In the problems we solve, the greatest cost, by far, is in the solution of the linear system of equations rather than in the assembly/updating.

We compare the fracture paths obtained by different criteria for problems consisting of multiple cracks and verify that, with mesh refinement, both criteria converge to the same fracture path provided the criterion for growth is the same. However, the convergence rate of the energy minimization technique to the converged crack path is found to be greater compared to that of the maximum hoop stress criterion. It is found that the converged fracture path lies in between the fracture paths obtained by each criterion for coarser meshes. This presents an opportunity to estimate an upper and lower bound of the true fracture path as well as an error on the crack path. Future work involves optimization of fracture increment lengths as well as the growth directions simultaneously.

REFERENCES

[1] R. J. Nuismer. An energy release rate criterion for mixed mode fracture. *International journal of fracture*, 11(2):245–250, 1975.

[2] K Hayashi and S Nemat-Nasser. Energy-release rate and crack kinking under combined loading. *Journal of Applied Mechanics*, (September 1981):520–524, 1981.

[3] B L Karihaloo, L M Keer, and S Nemat-Nasser. Crack kinking under nonsymmetric loading. *Engineering Fracture Mechanics*, 13(4):879–888, 1980.

[4] Fazil Erdogan and G C Sih. On the crack extension in plates under plane loading and transverse shear. *Journal of basic engineering*, 85:519, 1963.

[5] P.O. Bouchard, F. Bay, and Y. Chastel. Numerical modelling of crack propagation: automatic remeshing and comparison of different criteria. *Computer Methods in Applied Mechanics and Engineering*, 192(35-36):3887–3908, August 2003.

[6] M A Hussain, S L Pu, and J Underwood. Strain Energy Release Rate for a Crack Under Combined Mode I and Mode. *Fracture analysis*, 560:1, 1974.

[7] T Belytschko and T Black. Elastic crack growth in finite elements with minimal remeshing. *International Journal for Numerical*, 620(July 1998):601–620, 1999.

[8] Nicolas Moës, John Dolbow, and T E D Belytschko. A finite element method for crack growth without remeshing. 150(February):131–150, 1999.

[9] E. Budyn, G. Zi, N. Moes, and T. Belytschko. A method for multiple crack growth in brittle materials without remeshing. *International Journal for Numerical Methods in Engineering*, 61(10):1741–1770, November 2004.