

NUMERICAL STUDY OF TEMPERATURE AND STREAMFUNCTION PATTERNS BEFORE FULL CONVECTION IN GEOTHERMAL CELLS OF BÉNARD TYPE

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Under certain geometry and boundary conditions, coupled fluid flow and heat transfer processes in porous media between horizontal layers heated from below and cooling from above, typical theoretical geothermal scenarios, give place to organized structures of a same length – named Bénard cells – that repeat along the large horizontal domain [1, 2]. When the onset of convection is reached, the initial motionless system starts to move and a pattern of 2-D rolls – convection cells – in which neighbor cells have a different rotation, can be clearly distinguished. However, if the convection is not triggered, the system remains in the static no-flow situation set up before heating. We are interested in this work with flow patterns under the onset of convection. Figure 1 shows the physical scheme of the problem.

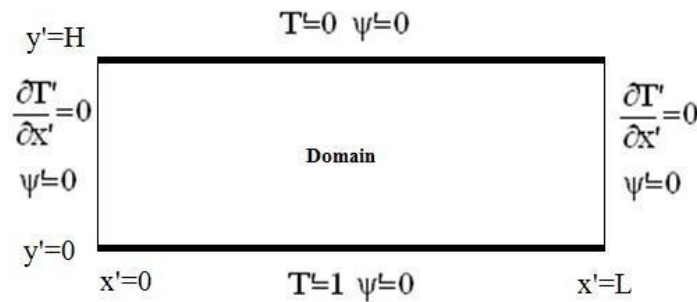


Figure 1. Geometry of the problem

Mathematical model is formed by the flow (u and v are the expressions of the horizontal and vertical components of the flow velocity) and energy equations plus those related to boundary conditions – Boussinesq approximation is assumed. These are:

$$u = -\left(\frac{K_x}{\mu}\right)\left(\frac{\partial P}{\partial x}\right), \quad v = -\left(\frac{K_y}{\mu}\right)\left\{\left(\frac{\partial P}{\partial y}\right) + \rho g\right\} \quad (1)$$

$$(\rho_f c_{p,f})u \left(\frac{\partial T}{\partial x}\right) + (\rho_f c_{p,f})v \left(\frac{\partial T}{\partial y}\right) = k_{m,x} \left(\frac{\partial^2 T}{\partial x^2}\right) + k_{m,y} \left(\frac{\partial^2 T}{\partial y^2}\right) \quad (2)$$

$$T_{(x,y=0)} = 1, T_{(x,y=H)} = 0, \left(\frac{\partial T}{\partial x}\right)_{(x=0,y)} = \left(\frac{\partial T}{\partial x}\right)_{(x=L,y)} = 0 \quad (3)$$

$$\Psi_{(x,y=0)} = \Psi_{(x,y=H)} = \Psi_{(x=0,y)} = \Psi_{(x=L,y)} = 0 \quad (4)$$

In these equations T and ψ are the dependent variables temperature ($^{\circ}\text{C}$) and streamfunction (m^2/s), respectively, P (N/m^2) the pressure, K (m/s) is the hydraulic conductivity, ρ (kg/m^3) the fluid density, g (m/s^2) the gravitational acceleration, μ the fluid viscosity ($\text{kg m}^{-1}\text{s}^{-1}$), $c_{p,f}$ ($\text{J}/\text{Kg}^{\circ}\text{C}$) the specific heat, k the thermal conductivity ($\text{W}/\text{m}^{\circ}\text{C}$); finally, x (m) and y (m) are the independent variables. The only dimensionless parameter that characterizes this problem in isotropic porous media is the so-named Rayleigh number

$$\text{Ra} = \left(\frac{Kg\Delta\rho H}{\mu\alpha}\right) \quad (5)$$

The onset of convection takes place for $\text{Ra} \approx 40$, so that we are interested in patterns resulting from $\text{Ra} < 40$, in order to investigate the ‘apparent’ motionless of the flow assumed by most researchers, its potential structuration in cells and the distortion of the temperature pattern. Despite the order of magnitude of the flow is clearly below to the values resulting for $\text{Ra} > 40$, where the convection has been triggered, the apparent motionless of the flow that predict the regular distribution of temperature horizontal isolines does not emerge. Negligible local changes in temperature produce instabilities that force the fluid to move in potential emergent cells that are more apparent as Ra increases. Viscosity forces balances with density driven forces but, in fact, both forces exist whatever the value of Ra .

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