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VARIATIONAL PRINCIPLES FOR CARDIAC ELECTROPHYSIOLOGY

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ABSTRACT

The numerical simulation of the electrical activity of the heart has experienced tremendous advances in the last decade. However, the acceptance of computational methods by the medical community and the translation of these tools to the clinic will largely depend on their reliability, efficiency and robustness. In this work, we exploit the variational structure first noted in [1] and present a minimax variational principle for the general time-discretized electrophysiology equations. Using results from variational analysis [2], we derive bounds on the time-step size in order for the semi-discrete problem to have a unique solution. We further show conditions under which the minimax problem is equivalent to an effective minimization principle, which is amenable to Rayleigh-Ritz finite-element analysis [3]. The time-step bounds guarantee the strict convexity of the objective function, thus ensuring the convergence of gradient-descent methods in the solution of the resulting nonlinear system. The proposed theory is applied to the FitzHugh-Nagumo model, which is shown to conform to the variational framework proposed in this work. The proposed framework and its numerical implications on the robustness of computer simulations are demonstrated by way of numerical examples of single-cell action potentials and 3D biventricular activation-repolarization sequences.

REFERENCES

[1] Colli Franzone, P. and Savaré, G. (2002). Degenerate evolution systems modeling the cardiac electric field at micro and macroscopic level. *Evolution equations, semigroups and functional analysis*, 1-26.

[2] Ekeland, I. and Témam, R. (1999) Convex Analysis and Variational Problems. SIAM Philadelphia

[3] Radovitzky R. and Ortiz, M. (1999) Error estimation and adaptive meshing in strongly nonlinear dynamic problems. *Computer Methods in Applied Mechanics and Engineering* 172, 203-240.