11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A MIXED HYBRID FINITE ELEMENT METHOD FOR THE COUPLING STOKES-DARCY FLOW

Iury Igreja[1], Cristiane O. Faria [1] and Abimael F. D. Loula [1]

[1] Laboratório Nacional de Computação Científica - LNCC/MCTI,
Av. Getúlio Vargas, 333, Quitandinha, Petrópolis/RJ - CEP: 25651-075,
{iuryhai, cofaria, aloc}@lncc.br

Key words: *Coupled Stokes-Darcy flow, Beavers-Joseph-Saffman interface condition, Stabilized finite element, Mixed-hybrid methods.*

Numerical methods to simulate the flow of incompressible viscous fluids through permeable heterogenous media have been widely developed due to several applications in petroleum engineering (well and reservoir interaction), hydrology (coupled surface and groundwater flows), hydrogeology (flow in fractured porous medium), as examples. In particular, a well known problem is the coupling of the free-fluid with the porous medium governed by Stokes and Darcy equations, respectively, and a slip boundary condition (due to Beavers-Joseph-Saffman [1, 10]) on the interface between them. Standard finite element implementation [2], Lagrange multipliers to impose the interface condition [6] or iterative procedures associated with domain decomposition techniques [3, 4] can be applied to solve this coupled problem. To deal with each subdomain, Rivière and Yotov [9] present a locally conservative coupled method through the use of a Discontinuous Galerkin (DG) method for the Stokes flow with the dual mixed formulation using Raviart-Thomas elements for Darcy flow. An extension of this work is presented by Rivière [8] where DG methods are used in both subdomains. However, the practical utility of DG methods have been limited by their more complex formulation, computational implementation and much larger number of degrees-of-freedom.

In this context, we propose a new stabilized hybrid mixed finite element formulation for the coupled problem combining the formulations proposed by Núñez et al [7] and Igreja et al [5] for the Darcy and the Stokes subdomains, respectively, that preserves the main properties of the associated DG methods applying Lagrange multipliers to impose weakly continuity on each side of the elements. The global system is assembled involving only the degrees of freedom associated with the multipliers and the variables of interest can always be solved at the element level. Results of some numerical experiments illustrate the flexibility and the robustness of the proposed finite element formulation.

References

[1] G. Beavers and D. Joseph. Boundary conditions at a naturally permeable wall. *J. Fluid Mech.*, 30(1):197–207, 1967.

[2] M. Correa and A. Loula. A unified mixed formulation naturally coupling Stokes and Darcy flows. *Comput. Methods Appl. Mech. Engrg.*, 198:2710–2722, 2009.

[3] M. Discacciati. Iterative methods for Stokes/Darcy coupling, domain decomposition methods in science and engineering. 40, 2004.

[4] M. Discacciati and A. Quarteroni. Analysis of a domain decomposition method for the coupling of Stokes and Darcy equations. *Numerical Mathematics and Advanced Applications, ENUMATH2001, Springer, Milan*, (3-20), 2003.

[5] I. Igreja, C. O. Faria, and A. F. D. Loula. Hybridized mixed finite element method for the Stokes problem. *Iberian Latin American Congress on Computational Methods in Engineering (CILAMCE), 2013, Pirenópolis. Anais Proceedings*, 2013.

[6] W. J. Layton, F. Schieweck, and I. Yotov. Coupling fluid flow with porous media flow. *SIAM J. Numer. Anal.*, 40(6):2195–2218, 2003.

[7] Y. R. Núñez, C. O. Faria, A. F. D. Loula, and S. M. C. Malta. A mixed-hybrid finite element method applied to tracer injection processes. *International Journal of Modeling and Simulation for the Petroleum Industry*, 6(1):51–59, 2012.

[8] B. Rivière. Analysis of a discontinuous finite element method for the coupled Stokes and Darcy problems. *J. Sci. Comput.*, 22-23(479-500), 2005.

[9] B. Rivière and I. Yotov. Locally conservative coupling of Stokes and Darcy flows. *SIAM J. Numer. Anal.*, 42(1959-1977), 2005.

[10] P. Saffman. On the boundary condition at the surface of a porous medium. *Stud. Appl. Math.*, L(2):93–101, 1971.