11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

EMBEDDED SOLIDS OF ANY DIMENSIONS IN THE EXTENDED FINITE ELEMENT METHOD

F. Duboeuf*[1] and E. Béchet[1]

[1] LTAS - Aerospace and Mechanical Engineering Department, University of Liège
Chemin des chevreuils, 1, B-4000 Liège, Belgium
fduboeuf@ulg.ac.be

Key words: *Embedded Solids, Boundary Conditions, Extended Finite Element Method.*

In the context of the extended finite element method (X-FEM), the geometric representation is not necessarily conform. Solids are immersed in a bulk mesh of the same dimension. The domain boundaries are then converted into an implicit geometry described by Level Sets. Other representation can obviously be used.

The enforcement of boundary conditions on boundaries crossing the elements requires special attention. Adding Neumann type boundary conditions remains as straightforward as in the standard finite element method. However, the X-FEM prevents the use of nodal collocation to enforce Dirichlet boundary conditions because of non-fitted meshes.

Various approaches have been proposed in the literature, based on the penalty method or Lagrange multipliers. Among these, two solutions to the Ladyzhenskaya-Babuška-Brezzi stability problem can be distinguished: stable and stabilized methods.

Initially developed to impose Dirichlet type boundary conditions on edges of 2D solids [1,2], then extended to faces of 3D solids [3], neither of these methods is suitable for this type of constraint on the implicit edges of a 3D solid.
Moreover, considering only solids of the same dimension as the mesh is overly restrictive. Why not immerse curves and surfaces in a mesh of higher dimension? For instance, in the case of fluid-structure interaction where the solid may just be a thin shell.

The purpose of this talk is to explore these two challenges:
First, we propose a new methodology to enforce Dirichlet boundary conditions in the X-FEM, available for every combination of the space and boundary dimensions. To define a stable Lagrange multiplier space, a new generalized vital vertex algorithm is based on concepts suitable for each dimension. Convergence analyses were performed on academic problems to validate the process.

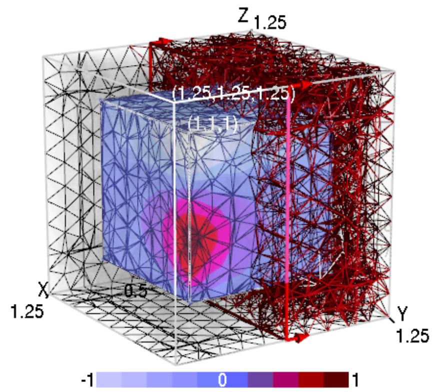

Figure 1: Solid under thermal loading: 1D Dirichlet boundary condition.

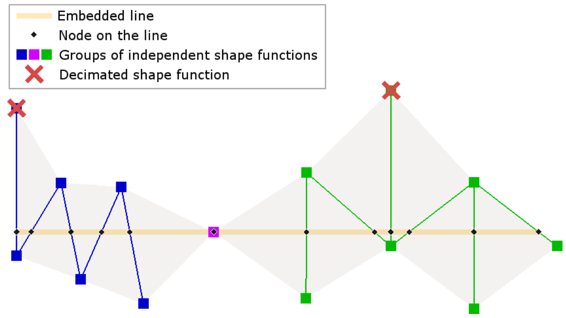

Figure 2: Reduction of the function space along a line: identify groups of independent shape functions and decimation.

Second, we examine how to build a function space to solve a typical problem along an implicit line immersed in a 2D mesh. Due to the mismatch between the line and the edges of the mesh, the number of shape functions associated with the mesh nodes needs to be reduced before using their trace along the 1D solid. The space obtained with the reducer algorithm previously used for the enforcement of boundary conditions, is too depleted to be able to represent a linear field, therefore it does not pass the patch test.

To tackle this problem, we introduce a new algorithm to define a reduced function space over an embedded solid of arbitrary dimension ($d \leqslant 2$). Several numerical examples illustrate the ability of this method to solve problems in 1 and 2 dimensions, that are embedded in a 2 or 3 dimensional background mesh.

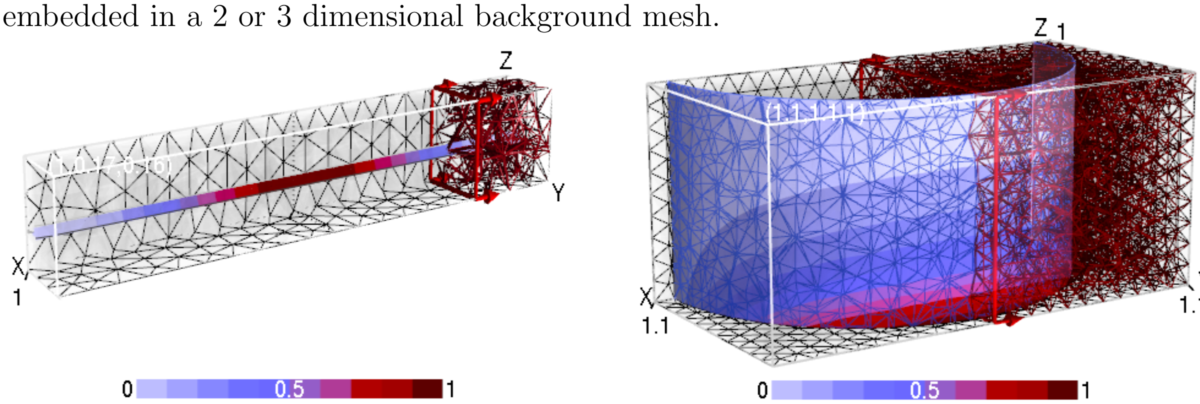

Figure 3: Embedded approximations along a line (left) and a surface (right) to the solution of thermal problems over 3D meshes.

The combination of both algorithms allows to treat any embedding i.e. 1, 2 and 3D problems embedded in 2 or 3D background meshes.

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