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Pseudo Arc-Length Method with Moving Mesh for Shock Wave Propagation

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A pseudo arc-length method (PALM) is proposed for numerical simulation of shock wave propagation. This method passes the discontinuities and establishes adaptive moving meshes in the physical space through introducing the arc-length parameter and transforming the computational domain. The key idea of this method comes from the original arc-length method through which the critical points are passed by tracing nonlinear equilibrium path in the arc-length space. For the shock wave propagation problem, we changes the governing equation in original physical space to the arc-length space and then solves the system in the arc-length space, finally converts the variables back to the physical space. In this process, the strictly discontinuous jump in the physical space is transforming into a smooth curve in the arc-length coordinates, and then the singularity of the solution in the original problem is eliminated or reduced.

The phenomena of shock wave propagation are always governed by hyperbolic conservation laws, and mathematically represented as time-dependent, often highly nonlinear systems of partial differential equations, thus a description of PALM for hyperbolic conservation laws is provided, including details of the procedure to deduce the conservation laws of 1D, 2D respectively. Notice that introducing the are-length parameter does not change the essence of partial differential equations, but bypass the shock discontinuous of physical space in transformed computational domain. Moreover, the process to solve the governing equations in the arc-length space is equivalent to solve the conservation laws in the physical space. Therefore, we refer to this method as “the pseudo arc-length method (PALM)”

In order to verify the feasibility of our proposed PALM, we apply the numerical algorithm to one- and two-dimensional Euler equations. Consider the Sod’s and Lax’s classical tube problem in 1D as the first example, and the second verification test for the 2D algorithm is conducted for the explosion problem.

The numerical results are shown in Figs.1 (a) and (b) corresponding to Sod’s and Lax’s problem. The shock and rarefaction waves are well resolved, and quite a number of grid points are moved to the region where the solution is nearly singular. It is obvious from Fig.1 that our approach behaves clearly better than direct finite volume method in dealing with contract problems. Both dissipation and numerical oscillation is reduced with the PALM, and the computational results well agree with exact solutions near the head and tail of shock occurs.

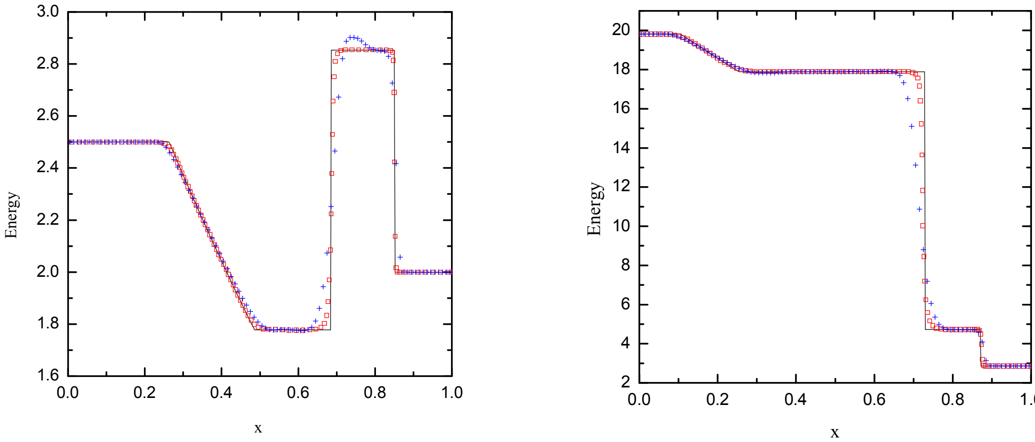

Fig.1. Sod problem and Lax problem: numerical results. “□”, “+” denote the numerical solution of the PALM and direct FVM, respectively, solid lines show the exact solutions.

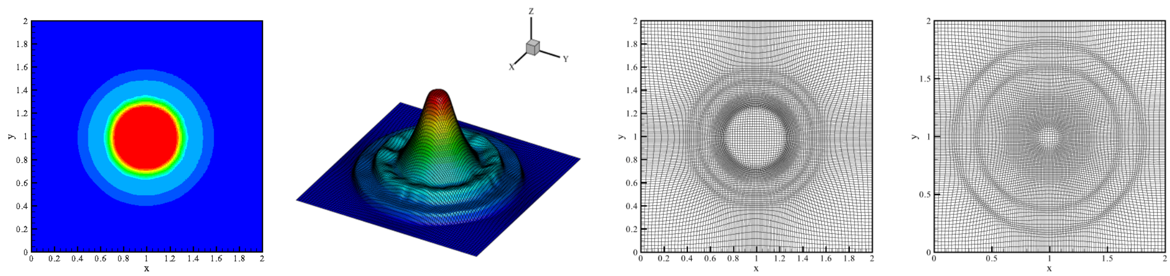

Fig.2. Explosion test density distribution obtained with PLAM (left) and corresponding moving mesh configuration (right).

Fig. 2 shows the numerical results of explosion problem. A circular shock wave and contract surface travel away from the centre and a rarefaction wave travels towards the centre. It can be observed that our method can establish adaptive moving meshes under circular initial conditions, and this test gives evidence that our method can be applied to simulate the explosion problem and shock waves propagation.

In summary, a new method is proposed to solve the hyperbolic conservation laws with shock waves. Numerical experiments of Sod problem and explosion problem demonstrate that this approach generates solution with high resolution and adaptive moving mesh, and the computation is more robust and efficient than classical discrete methods in capturing and tracking shock discontinuity. As a new approach, the PALM can be widely developed to practical shock waves problems in science and engineering.

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