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Nonlinear dynamics of a two-layer composite beam with nonlinear interface with different boundary conditions

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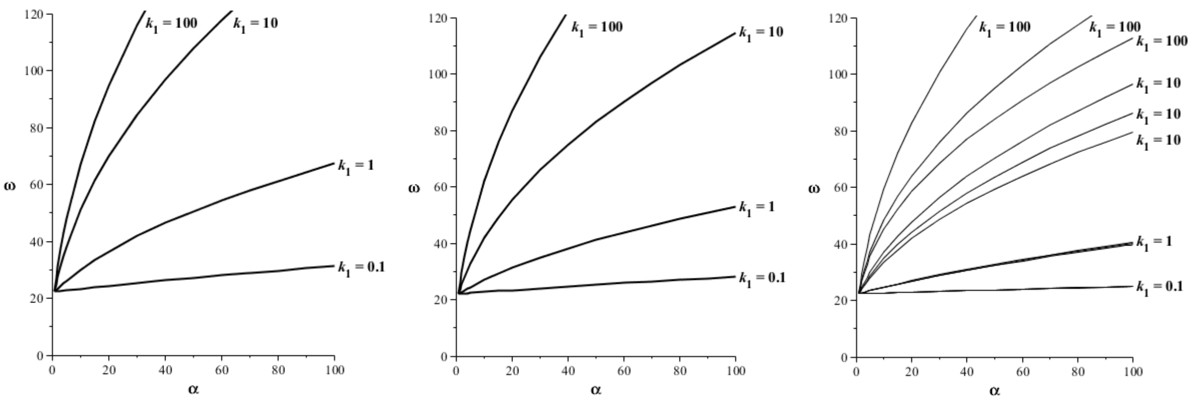
The dynamic behavior of two-layer beams has been largely studied in the literature, where numerical, analytical/theoretical as well as experimental studies can be found. To the authors' knowledge, the existing analyses focus on the nonlinearities of the beams/plates, and not explicitly on the nonlinearity of the interlayer, which is instead the main assumption of this work.

We extend the work reported in [1] where a free-free two-layer composite beam with nonlinear viscoelastic zero-thickness interface is considered. The beams have an Euler-Bernoulli kinematics and perfect adherence in the normal direction: only slipping is allowed at interface. The reliability of these hypotheses in the linear regime has been discussed in [2, 3] where the full problem with shear deformations, axial and rotational inertia, and interface uplift is considered, also with different boundary conditions in [4]. The beams' cross-section are not necessary rectangular, and the results can be apply to a variety of engineering problems, like for example structural glass, cross-ply laminated composted beams, steel concrete beams, concrete/steel/wood beams reinforced with FRP sheet.

The applications which are lurking in the background not only have a small thickness, but they also have an interlayer which is made of a material which is less stiff than that of the beams. In this situation, the interlayer can behave nonlinearly, still remaining in the elastic regime, while the beams behave linearly. To give an idea of the nonlinear elastic behavior of the interface we note that Ivanov et al. [5] obtain $\tau=0.5173\gamma+0.0772\gamma^3$ (the shear stress τ is in [MPa], γ is the shear strain) in static experiments of PVB, which is the interlayer commonly used in structural glass.

In this work we also pay attention to the effects of different boundary conditions on the nonlinear frequencies of two-layer beam. We consider both symmetric (free-free, fixed-fixed and hinged-hinged) and non-symmetric (free-fixed and hinged-fixed) b.c. We also consider the case of layers having different b.c., one fixed-fixed and the other free-free, which occurs for example in coating and in beams reinforced with FRP strips.

The free vibrations are studied by means of the multiple time scale method, which permits us to address the nonlinear problem analytically and provides an accurate estimation of the nonlinear, amplitude dependent, natural frequencies. The influence of damping and external excitation are considered by the same mathematical tool. The first order terms of the asymptotic expansion provide the linear natural frequencies, which are found to depend on two dimensionless parameters only and, for boundary conditions different on each layer, also on the ratio between the axial stiffnesses of each layer (Fig. 1). The second order terms vanish by symmetry, while the third order terms provide the nonlinearity coefficients measuring how the natural frequencies depend (nonlinearly) on the vibration amplitude. As the linear frequencies, also these coefficients depend on two dimensionless parameters and, for boundary conditions different on each layer, also on the ratio between the axial stiffnesses of each layer.



a) b) c)

Figure 1. The first natural frequency ω for different b.c. a) free-free, b) free-fixed and c) corresponds to free-free layer 2 and fixed-fixed layer 1. In the case c), for each value of k_1 there are three curves (they are barely visible for $k_1 = 1$ and for $k_1 = 0.1$): the upper curve corresponds to $E_1A_1 = 0.01E_2A_2$, the medium one to $E_1A_1 = E_2A_2$ and the lower one to $E_1A_1 = 100E_2A_2$.

The results of the present analysis permit quantitative determination of the amplitude of the vibrations when the amplitude of the excitation is known for different boundary conditions.

REFERENCES

[1] S. Lenci, J. Warminski, Free and forced nonlinear oscillations of a two-layer composite beam with interface slip, *Nonlinear Dyn.*, Vol. **70** (3), pp. 2071-2087, 2012.
[2] S. Lenci, F. Clementi, Effects of shear stiffness, rotatory and axial inertia, and interface stiffness on free vibrations of a two-layer beam, *J. Sound Vib*., Vol. **331**, pp. 5247-5267, 2012.
[3] S. Lenci, G. Rega, An asymptotic model for the free vibrations of a two-layer beam, *Eur. J. Mech. A/Solids*, Vol. **42**, pp. 441–453, 2013.
[4] S. Lenci, F. Clementi, On flexural vibrations of shear deformable laminated beams, *ASME 2012 - International Mechanical Engineering Congress & Exposition*, Vol. 4, pp. 581-590, Houston, USA, 2012.
[5] I.V. Ivanov, D.S. Velchev, T. Sadowski, M. Kneć, Computational models of laminated glass plate under transverse static loading, in: Altenbach, H., Eremeyev, V. (eds.) Shell-Like Structures Non-classical Theories and Applications. Advanced Structured Materials, Vol. **15**, pp. 469–490. Springer, Berlin, 2011.