Finite Element Method for a Slit Model with Damping of Air Viscosity

*Manabu Sasajima¹, Takao Yamaguchi² and Yoshio Koike¹

¹ Foster Electric Co., Ltd., 1-1-109, Tsutsujigaoka, Akishima, Tokyo, 196-8550 Japan,
  sasajima@foster.co.jp http://foster.co.jp/

² Gunma university, 1-5-1 Tenjin-cho, Kiryu, Gunma, 376-8515 Japan, yamagume3@gunma-u-ac.jp
  http://www.tech/gunma-u.ac.jp/

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1. Introduction
   The conventional acoustic analysis approach is used predominantly for relatively large structures or large equipment. For structures with a small volume, such as the occluded-ear simulator (IEC60318-4) for the measurement of insert-type earphones, very few methods of sound propagation analysis are available. This ear simulator has very narrow pathways to control the acoustic resistance; specifically, the speed of sound decreases and a phase delay occurs. Therefore, to perform accurate acoustic analysis of small devices, the effect of air viscosity should be considered. This effect is not considered in conventional acoustic analysis. In the present study, we developed a new FEM that considers the effects of air viscosity in narrow portions of the sound pathways in small devices. This method was developed as an extension of the acoustic FEM proposed by Yamaguchi [1, 2] for a porous sound-absorbing material. We attempted numerical analysis in the frequency domain using our acoustic solver, which utilizes the proposed FEM. For the numerical calculations, we used tube models with slit cross sections. Then we compared the results obtained by the proposed FEM with those obtained by theoretical analysis[3] and the conventional FEM.

2. Numerical procedures
   The viscosity energy $\vec{D}$ of a viscous fluid can be expressed as follows:
   \[
   \vec{D} = \frac{1}{2} \oint \frac{1}{2} \left\{ \vec{\Gamma} \right\} dx dy dz
   \]
   where $\left\{ \vec{\Gamma} \right\}$ is the stress vector attributable to viscosity. $\left\{ \vec{\Gamma} \right\}$ is the strain vector. At a result, we can obtain the following discretized equation of an element by using Lagrange’s equations:
   \[
   -\omega^2 [M_e] \{u_e\} + [K_e] \{u_e\} + j\omega [C_e] \{u_e\} = \{f_e\}
   \]
   We use $\{\bar{u}_e\} = j\omega \{u_e\}$ in this equation because a periodic motion having angular frequency $\omega$ is assumed. $[M_e]$, $[K_e]$, $[C_e]$, and $\{f_e\}$ are the element mass matrix, element stiffness matrix, element viscosity matrix, and nodal force vector, respectively.

3. Calculation
   To verify our method, we carried out an acoustic damping analysis for slits using 3D FEM. A model is a 1/4 solid model symmetrical about the x-z plane and the x-y plane. The width of the model was 2.0 mm, the height was 0.5 mm, and the length was 16.6 mm. This model used tetrahedral elements having four nodes. We analyzed the frequency responses using the
proposed FEM and compared the results with those of the above-described theoretical method that considers the viscosity and those of the conventional FEM that does not consider the attenuation. Fig.1 compares the analytical results for models. We determined the effect of damping on the results obtained by the proposed FEM and the theoretical method. The results obtained by the conventional FEM do not exhibit attenuation of the resonance peaks. In addition, it can be seen that the results obtained by the proposed FEM and the theoretical method are approximately same. Fig.2 shows the relative error between the proposed method and the theoretical method. Numbers (3, 5, 7, 9, and 10) are the number of element divisions in each of the height. Over all frequency bands from 5,000HZ to 25,000 Hz, the relative error is less than 1% for 5 divisions or more. Further, the numerical solution can be confirmed to converge to the analytical solution with increasing number of component divisions.

4. Conclusion

We developed a new acoustic FEM that considers the damping effects of the viscosity of air. We compared the sound pressure versus frequency characteristics obtained by the proposed FEM with those obtained by the theoretical method and the conventional FEM. The comparison showed that the obtained results are in good agreement. The proposed acoustic FEM was therefore confirmed to have good analytical accuracy. It was also confirmed that the numerical results of the proposed method converged to the theoretical solutions. It was therefore possible to confirm the effectiveness of the approach.

REFERENCES