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STABILIZED GALERKIN METHODS FOR THE ADVECTION OF DIFFERENTIAL FORMS WITH DISCONTINUOUS VELOCITY FIELDS

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In view of solving the problem of resistive magnetohydrodynamics (MHD), a numerical treatment of the equations governing the evolution of electromagnetic fields based on finite element exterior calculus (FEEC) is proposed. The MHD equations are a widely used macroscopic model for the behavior of conducting fluids, like plasma, interacting with electromagnetic fields. Due to the strongly non-linear hyperbolic character of the system, the inevitable shock formation introduces discontinuities into the velocity field. For this reason, the quasi magneto-static Maxwell's equations in the limit of small diffusivity are of specific interest in the case of given discontinuous transport velocities.

In this context, the more general transport problem for a differential k-form is numerically tackled with implicit time integrators and structure preserving spatial discretizations relying on discrete differential forms. In particular, for the magnetic advection problem we formulate a stabilized Galerkin finite element method based on $\mathrm{H}(\mathbf{curl})$-conforming edge elements. The numerical analysis of the method is quite challenging on account of lacking well-posedness results for the continuous problem when $k \geq 1$. The scheme we propose is an extension of the Eulerian conforming stabilized Galerkin method introduced in [1] for stationary magnetic advection with Lipschitz continuous velocity fields.

Numerical examples in 2D on both structured and unstructured meshes for the steady-state and transient advection problem with discontinuities in the velocity are provided.

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