

# **Assessing numerical viscosity for Implicit LES using linear instability analysis: examples for a high order incompressible Discontinuous Galerkin – Fourier solver**

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## **ABSTRACT**

This work presents a novel approach to quantify numerical viscosity for Implicit LES calculations, which is based on linear stability analysis. The underlying idea of the method is to compute the exponential growth of small perturbations (e.g. random noise) before nonlinearities take over (i.e. saturated regime). An increase in numerical viscosity relates to a decrease in the perturbation growth rate, hence making possible to establish bounds on the numerical viscosity added by a particular mesh and the underlying numerics.

We apply this technique to a high order (order  $> 3$ ) Discontinuous Galerkin – Fourier incompressible Navier-Stokes solver developed by the first author [1-4]. Using the abovementioned technique, we compare the numerical viscosities of two different implementations of the nonlinear terms showing that numerical viscosity enhances the solver stability a high Reynolds numbers. Furthermore, since unstructured hybrid meshes (tri-quads) may be used together with a Fourier extension in the third direction, the present method permits the evaluation of spatially varying numerical viscosity introduced by the different nonlinear formulations. The quantification of numerical viscosities sets the basis for future control strategies that may be applied in the context Implicit Large Eddy Simulation technique.

Various cases for three dimensional flows aft NACA airfoils are presented for turbulent flow conditions, as shown in figure 1.

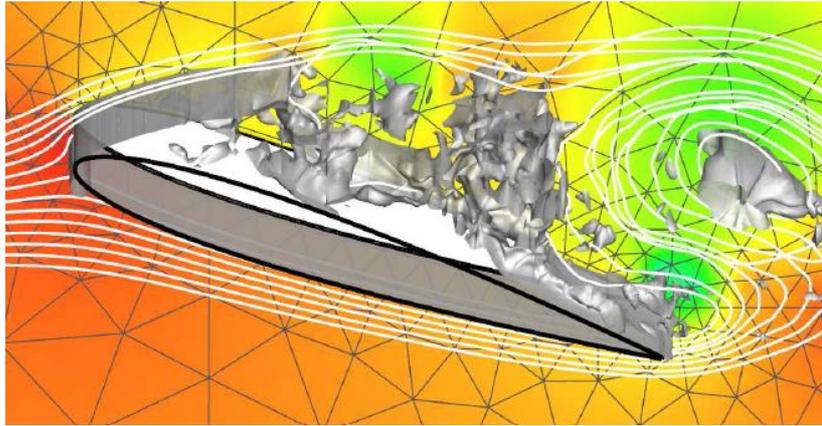


Figure 1: Implicit LES high order DG-Fourier solutions aft a three dimensional NACA0012 at  $Re=10000$  and  $AOA=20^\circ$  using polynomial order 5 and 16 Fourier planes.

## References

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