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ENTROPY CONSERVATIVE AND ENTROPY STABLE FINITE VOLUME/FINITE ELEMENT SCHEMES FOR THE NAVIER-STOKES EQUATIONS ON UNSTRUCTURED MESHES

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To solve flow problems associated with the Navier-Stoles equations, we construct a mixed finite volume/finite element for the spatial approximation of the convective and the diffusive parts of the flux, respectively. The finite volume component of the method is adapted from the authors' construction for hyperbolic conservation laws and unstructured triangular grids [3], of two-dimensional finite volume extensions of the entropy stable schemes [7]. We develop a novel energy preserving second-order accurate scheme that is very simple to implement, is computationaly cheap and is stable compared to other existing energy preserving schemes [2]. To allow for a correct dissipation of energy in vicinity of shocks, a novel numerical diffusion operator of the Roe type [5] is designed. The energy preserving scheme, together with this diffusion operator, gives an energy stabe scheme for Euler equation on unstructured grids. We apply a standard reconstruction procedure to obtain a second-order accurate scheme. For the viscous terms, we use a centered finite element approximation [6]. The explicite three-stage third-order Runge-Kutta (RK3) [4] method will be used. To improve the quality of the resolution, we use a grid adaptation proposed by [1]. Numerical experiments in two space dimensions are presented to demonstrate the robustness of the proposed schemes. Numerical experiments include, Sod shock tube problem, vortex advection, supersonic viscous flow over a flat plate and viscous flow past a NACA0012 airfoil and comparison with other methods, lead to fairly competitive results with favorable computing times and very sharp capture of shocks.

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