

COMBINED DOMAIN DECOMPOSITION AND MODEL ORDER REDUCTION METHODS FOR THE SOLUTION OF COUPLED AND NON-LINEAR PROBLEMS

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Key Words: *Multiphysics Problems, Model Order Reduction, Proper Orthogonal Decomposition, Domain Decomposition, Microsystems, MEMS.*

Design and reliability assessment of *Micro Electro Mechanical Systems* (MEMS) are based on realistic simulations of complex multi-physics (electro-mechanical, thermo-mechanical, magneto-mechanical...) and/or highly non-linear and irreversible processes (elasto-plastic, damage, fracture...) which must be solved with sufficient accuracy. This kind of coupled and non-linear problems lead to numerical models with a large number of degrees of freedom, which are prohibitive to solve in a time duration compatible with the design process making use of standard finite element strategies. These applications call for approximation techniques that replace large-scale computational models by simpler ones capable of reproducing their essential features at a fraction of their computational cost.

Recently, the Authors proposed the coupled use of Domain Decomposition (DD, [1]) methodologies together with Model Order Reduction (MOR) techniques, based on the use of Proper Orthogonal Decomposition (POD, [2]) to reduce the computational time of simulations without reducing the accuracy and at the same time to efficiently exploit the possibilities offered by large scale and parallel computing. Applications in the case of the electro-mechanical coupled problem for microsystems have been published in [3]-[5].

In the case of coupled problems, starting from a reformulation of the DD method of [1], in which the decomposition is viewed in a physical sense (i.e. the multi-physics of the problem automatically defined from the geometry of the decomposition), an enhancement of this technique was proposed, by further decomposing the mechanical domain, enforcing displacement continuity between the initially decoupled sub-domains and introducing various strategies for the application of POD to the mechanical part of the solution domain.

In the present work, the authors propose a new strategy, which is part on recent advances in model order reduction strategies for highly nonlinear structural problems like e.g. plasticity and fracture (see e.g. [6]), that exploits the potentialities of the coupled use of DD and POD methods. An elastic-plastic structural dynamic problem is considered. To represent the structural behaviour beyond the onset of plastic strain localization, in the present work the POD reduced basis is *adapted* in two different ways: first, during the training part of the simulation (i.e. when the basis is constructed), the reduced space is updated as soon as a new snapshot is collected. Secondly, an *on-line* adaptation technique of the reduced space is performed, through a *behavior check* during the reduced analysis. In those parts of the

structure in which the non-linear phenomena occur, the POD reduced analysis stops, and, through a zoom-in strategy (i.e. downscaling), the non-linear modeling is performed. The *rich non-linear* and *reduced linear* regions are processed simultaneously and are glued together through interface relations.

As an example, Figure 1 shows the elastic-plastic response of a two-dimensional structural frame with non-linear elastic-perfect plastic constitutive behavior. The coupled use of POD and DD allows minimizing the computing time, without reducing the accuracy, and a computational gain amounting up to 50 % can be attained.

The proposed technique is one step further in the formulation of a general DD-POD approach which allows for the simulation of multi-physics and/or highly non-linear coupled problems, in the presence of irreversible material behaviors like e.g. plasticity and fracture.

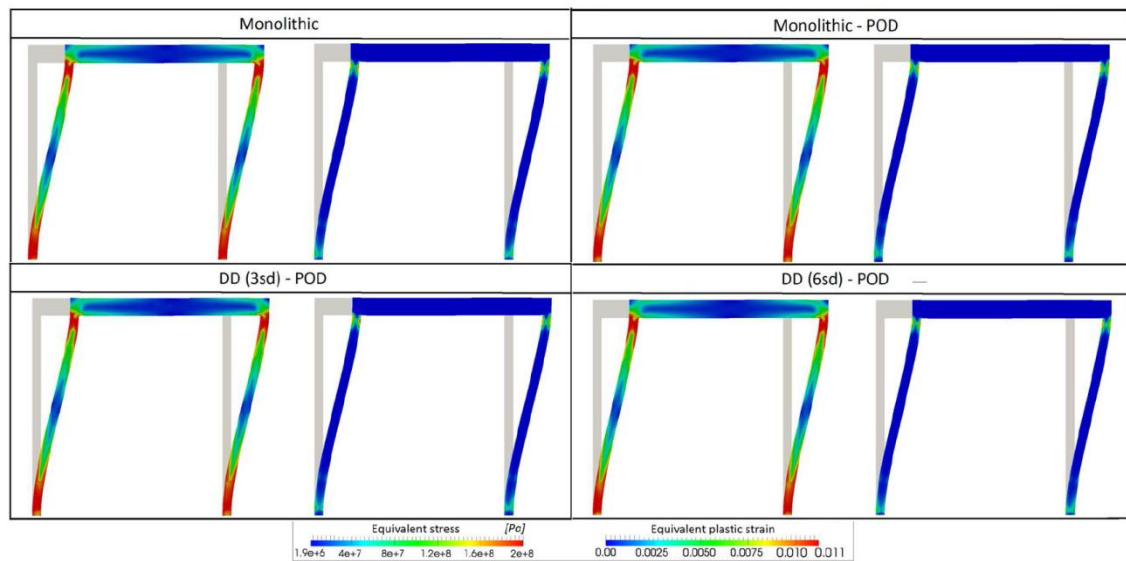


Figure 1: von Mises equivalent stress and equivalent plastic strain on the deformed configuration (amplification factor equal to 5). Comparison between the reference monolithic solution and the proposed methodology.

REFERENCES

- [1] A. Combescure and A. Gravouil, Multi-time-step explicit-implicit method for non-linear structural dynamics. *Int. J. Numer. Methods Eng.*, Vol. **50**, pp. 199-225, 2001.
- [2] G. Kerschen, J.C. Golinval, A. Vakaris, L.A. Bergman, The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical system: an overview. *Nonlinear dynamics*, Vol.**41**, pp. 147-169, 2005.
- [3] F. Confalonieri, A. Corigliano, M. Dossi, M. Gornati, A domain decomposition technique applied to the solution of the coupled electro-mechanical problem. *Int. J. Numer. Meth. Engng.* Vol. **93** (2), pp. 137-159, 2013.
- [4] Corigliano, M. Dossi, S. Mariani. Domain decomposition and model order reduction methods applied to the simulation of multiphysics problems in MEMS. *Computers & Structures*, Vol. **122**, pp.113-127, 2013.
- [5] A. Corigliano, M. Dossi, S. Mariani. Recent advances in computational methods for microsystems. *Advanced materials research*, Vol. **745**, pp. 13-25, 2013.
- [6] P. Kerfriden, O. Goury, T. Rabczuk, S.P.A. Bordas, A partitioned model order reduction approach to rationalize computational expenses in nonlinear fracture mechanics. *Computer Methods in Applied Mechanics and Engineering*, Vol. **256**, pp. 169-188, 2013.