A TWO STEP PROCESS FOR SHAPE OPTIMIZATION IN COMPUTATIONAL FLUID DYNAMICS

E. Betancur V\textsuperscript{1,*}, Ch. Dapogny\textsuperscript{2}, P. Frey\textsuperscript{2} and M.J. Garcia\textsuperscript{1}

\textsuperscript{1} Mecanica aplicada, Universidad EAFIT, Medellin, Colombia, \{ebetanc2,mgarcia\}@eafit.edu.co
\textsuperscript{2} Laboratoire J.L. Lions, UPMC Univ Paris 06, Paris, France, \{dapogny,frey\}@ann.jussieu.fr

Key words: \textit{CFD, optimization, SIMP method, shape derivative, mesh adaptation, FEM}

Introduction

This work concerns a shape optimization problem, i.e. that of minimizing an objective function \( J(\Omega) \) of the domain variable \( \Omega \), in the context of computational fluid dynamics (CFD). We investigate a twofold approach, consisting in a preliminary use of the so-called SIMP (Solid Isotropic Material with Penalization) method, followed by an optimization method relying on the shape derivative. The numerical scheme has been implemented using \textit{FreeFem++}[4] and involves a mesh adaptation stage to improve the boundary approximation as well as the numerical accuracy and efficiency of the method.

1 First stage: topology optimization

The first step is based on a classical SIMP formulation with a material distribution model proposed by Khadra [3] and used first by Borrvall [2] in fluid optimization. A fictitious solid domain is mimicked by using a Brinkmann penalization of the Stokes equation, a heuristic based on the theory of porous media: a term \( \alpha u_i \) is added to the Stokes equation posed in each subdomain \( \Omega_i \) of the fixed computational domain \( \Omega \), which then reads:

\[
-\mu \Delta u_i + \alpha(\rho) u_i = f_i - \nabla p.
\]  

(1)

The density function \( \rho \) is defined over the entire domain \( \Omega \), and takes the value \( \rho = 1 \) (resp. \( \rho = 0 \)) on the fluid (resp. solid) part. The inverse permeability \( \alpha \) is defined as a function of the density \( \rho \), and accounts for a penalization parameter.

In order to reduce the “losses” of the Stokes system, the optimization function \( \Phi(\vec{u},\rho) \) is devised to minimize the power dissipation of the fluid. The Stokes problem (1) is endowed with classical Dirichlet boundary conditions. This yields the optimization problem: \textit{Find} \( (u,p) \) \textit{solution to Problem} (1) \textit{such that}:

\[
\min_{\rho} \Phi(u_i(\rho),\rho) = \int_{\Omega} \left( \mu \nabla u_i : e(u_i) + \alpha(\rho) ||u_i||^2 \right) \, dx,
\]

to which a volume constraint is added.
A two step CFD optimization process: topology and shape optimization

2 Second stage: shape optimization

The fluid domain boundary resulting from the previous stage is usually not very accurate (as it corresponds to an average isovalue of a density function). Hence, a ‘geometric’ shape optimization procedure based on an objective function $J(\Omega^0)$ of the fluid part $\Omega^0$ of the domain is carried out on an unstructured (adapted) mesh in order to improve its description (see e.g. [1], Chap. 6): the analysis of the shape derivative of $J$ makes it possible to compute a descent direction for $J$ from a given shape $\Omega^0$, as a vector field $V_{\Omega^0}$.

3 Results

The diffuser example (see [2]) is implemented to validate our approach (Fig. 1). The SIMP optimization process yields a resulting shape that is then further optimized using the shape derivative method. At completion, a smooth explicit boundary is obtained for the optimal design with respect to the objective function.

REFERENCES


