

## A SIMPLE RECOVERY-BASED ERROR ESTIMATOR FOR THE STABLE GENERALIZED FINITE ELEMENT METHOD (SGFEM)

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**Key Words:** *Error estimator, Superconvergent Patch Recovery, Singular Value Decomposition, Stable Generalized Finite Element Method, Conditioning*

A recovery-based a-posteriori error estimator for the stable generalized finite element method (SGFEM) [1, 2] is hereby presented aiming to assess the accuracy of this method towards polynomial enrichments. The SGFEM proposes, basically, a modification of the enrichments used in the GFEM [3, 4] seeking to improve the conditioning of the stiffness matrix. The linear dependences introduced by the polynomial enrichments decrease the conditioning of the system matrix, therefore justifying the choice by the SGFEM. The proposed estimator for the SGFEM is result of an extension to this method of the superconvergent patch recovery (SPR) technique [5, 6], which is widely used in the evaluation of recovered stress fields from the conventional finite element solutions. In this estimator the interpolation polynomials for recovered stress fields defined in each cloud (set of elements sharing the same node) are identified from superconvergent point values, by using the singular value decomposition (SVD) strategy [7]. The number of these points, which are coincident with the quadrature integration points, is defined on the basis of the degree of the complete polynomial approximation for the local field of enriched displacements. Due to the resulting combination between SPR and SVD techniques, the estimator is denoted as SPR/SVD. In the last step of the recovering procedure the partition of unity concept is explored to compute the values for recovered stresses at points in the domain overlapped by more than one cloud. In order to assess aspects such as efficiency and computational performance, the error energy norms obtained with the estimator and with the analytic solution are compared. Related to this aim, two benchmark problems are considered, both discretized by two-dimensional meshes. Finally, the global effectivity index is addressed, and based on it a brief comparison between GFEM and SGFEM is presented.

### REFERENCES

- [1] I. Babuška and U. Barnejee, Stable generalized finite element method (SGFEM). *Comput. Methods in Appl. Mech. and Engrg.*, pp. 201–204, 2012.
- [2] V. Gupta, C. A. Duarte, I. Babuška and U. Barnejee, A stable and optimally convergent generalized FEM (SGFEM) for linear elastic fracture mechanics. *Comput. Methods in*

- Appl. Mech. and Engrg.*, Vol. **266**, pp. 23–39, 2013.
- [3] C. A. Duarte, I. Babuška and J. T. Oden, Generalized finite element methods for three-dimensional structural mechanics problems. *Computer & Structures*, Vol. **77**, pp. 215–232, 2000.
- [4] T. Strouboulis, K. Copps and I. Babuška, The generalized finite element method. *Comput. Methods in Appl. Mech. and Engrg.*, Vol. **190**, pp. 4081–4193, 2001.
- [5] O. C. Zienkiewicz and J. Z. Zhu, The superconvergent patch recovery and a posteriori error estimates - part 1: the recovery technique. *International Journal for Numerical Methods in Engineering*, Vol. **33**, pp. 1331–1364, 1992.
- [6] O. C. Zienkiewicz and J. Z. Zhu, The superconvergent patch recovery and a posteriori error estimates - part 2: error estimates and adaptivity. *International Journal for Numerical Methods in Engineering*, Vol. **33**, pp. 1365–1382, 1992.
- [7] A. Quarteroni, R. Sacco and F. Saleri, *Numerical Mathematics*, Springer, 2000.