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DIRECT SIMULATIONS OF THE COMPRESSIBLE NAVIER-STOKES EQUATIONS FOR MULTIPHASE FLOWS

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Compressible multiphase and multicomponent flows are common in a variety of applications including cavitation dynamics and erosion, hyper-velocity impact, shock interaction with density variations, and combustion. Numerical simulations can provide a wealth of data that would be difficult to obtain otherwise, in which case an accurate description of material interfaces separating different fluids and/or phases and interactions with shock waves is necessary to understand the detailed physics of these flows. Shock-capturing approaches have been used to simulate these flows by solving the Euler equations; however, a naive implementation of shock capturing leads to spurious pressure oscillations, which may further trigger the growth of hydrodynamic instabilities [1,2]. The current interest lies in interface capturing, in which interfaces are regularized onto the numerical grid in a fashion analogous to shock capturing [1, 2, 3]. To prevent pressure oscillations, specific transport equations for variables entering the equation of state must be solved [2]. This approach has been used to simulate the multicomponent Euler equations and extended to high-order accuracy [3]. It has further been shown recently that temperature errors may be generated by interface-capturing schemes at material discontinuities; in some cases, relative errors on the order of 50% may occur [4]. Although this issue is not relevant for the Euler equations since temperature is a passive quantity, it is problematic for the Navier-Stokes equations for several reasons. The thermal conduction term may produce errors in the total energy, which then propagates to the pressure and other quantities due to temperature errors. Furthermore, an incorrect treatment of temperature may lead to erroneous surface tension and phase change processes, and may affect the heating of neighboring solids.

In this contribution, pressure and temperature errors are investigated for interface-capturing methods based on Weighted Essentially Non-Oscillatory (WENO) [5] methods and applied to gas-liquid problems. The emphasis lies in the compressible Navier-Stokes equations including viscous stresses and heat conduction. In our talk, we will show analytically and through numerical examples that both temperature and pressure errors can be prevented by solving additional transport equations for the parameters entering the equation of state in appropriate conservative or non-conservative forms. In a sample result, Fig. 1 shows temperature and pressure errors caused by the method of Shyue [2] and our proposed scheme related to the advection of a single two-dimensional air bubble in water. The flow moves at a constant velocity and the phases are in thermodynamic equilibrium (i.e., with the same pressure and temperature). Since surface tension and mass transfer are neglected, the bubble should simply advect, in dynamic and thermodynamic equilibrium with its surroundings. After a period of time, it can be seen that the interface has diffused numerically and, more importantly, that temperature errors at the interface using Shyue's scheme may be large (60%); these errors are propagated to all other fields. On the other hand, using the current scheme reduces the errors to the order of round-off. It should be noted that the method of Shyue was never intended for the Navier-Stokes equations; the motivation for using this scheme as a comparison is to highlight possible issues when directly applying current methods for the multicomponent Euler equations to the Navier-Stokes equations.

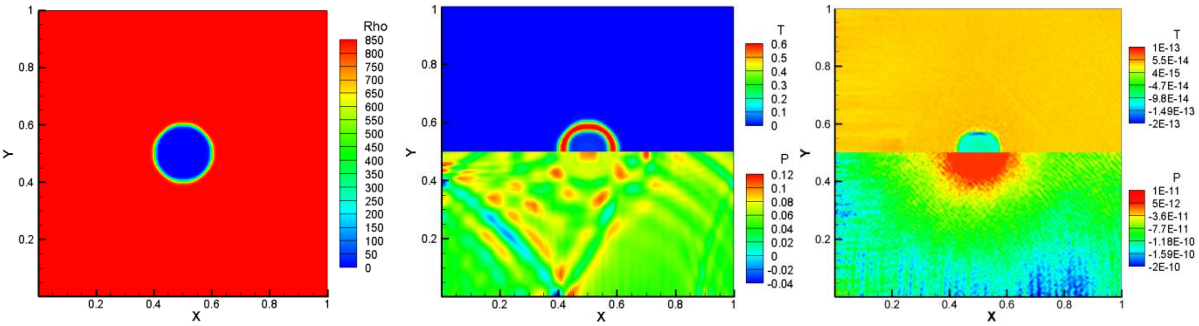

Figure 1. Bubble advection in wáter, a) contour of density, b) temperature errors (top) and pressure errors (bottom) for Shyue's scheme, c) temperature errors (top) and pressure errors (bottom) for current scheme

Although implementing shock- and interface-capturing schemes under the described conditions can prevent both temperature and pressure errors, excessive numerical dissipation may become problematic at late times, with the resulting computational cost preventing appropriate resolution of small-scale features. To address this problem, we are developing a solution-adaptive central/shock-capturing finite difference scheme for efficient computations of compressible multiphase flows [6]. In this approach, a discontinuity sensor discriminates between smooth and discontinuous regions; then, the appropriate form of central differences is derived for smooth regions and high-order WENO schemes are applied only at discontinuities. The standard conservative approach is followed for shocks and contacts, but material discontinuities are treated by reconstructing the primitive variables and solving the appropriate transport equations in conservative and/or non-conservative form to prevent temperature and pressure errors. The hybrid nature of the method allows for efficient and accurate computation of different types of discontinuities. Results from 2D simulations of non-spherical bubble collapse near a solid wall will be presented at the meeting.

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