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A NEW PARTICULAR SOLUTION STRATEGY FOR HYPERBOLIC PROBLEMS USING HYBRID-TREFFTZ FINITE ELEMENTS

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Trefftz finite elements [e.g. 1,2] approximate the solution of boundary value problems using functions that satisfy locally the homogeneous form of the domain equation (the Trefftz-compliance condition). The prescribed initial and boundary conditions, which are not satisfied, in general, by the trial functions, are enforced using collocation, least squares or Galerkin's method.

Due to the physical meaningfulness of the trial functions, hybrid-Trefftz elements work well in situations where conventional elements encounter difficulties. Indeed, studies have shown that problems involving stress concentrations, shear locking, high solution gradients and crack propagation can be handled without cumbersome mesh refinements by Trefftz elements [3]. Infinite domains, high frequency excitations, incompressible media and gross mesh distortions are also efficiently modelled [4]. Moreover, Trefftz-compliant trial functions reduce all terms in the finite element solving system to boundary integral expressions when the governing equation is homogeneous.

On the other hand, Trefftz-compliant bases fail to recover the solution of the non-homogeneous problems that occur, for instance, when initial conditions terms are present in the governing equations. In such situations, particular solution functions must be added to the Trefftz basis to model the effect of the source terms.

This issue has received significant attention in the last three decades, particularly in the Boundary Element Method context. Despite some variability, most of the approaches can be classified into two main categories: direct approaches, based on the evaluation of the particular solution using Green's functions, and indirect approaches (e.g. the Dual Reciprocity Method), where the particular solution is approximated using additional trial functions [5].

A new indirect approach for approximating the particular solution of non-homogeneous hyperbolic boundary value problems is presented in this paper. Unlike the Dual Reciprocity Method (DRM), which constructs approximate particular solutions using radial basis, polynomial or trigonometric functions, the method addressed here uses functions that satisfy locally the homogeneous form of the static problem obtained by discarding all time-derivative terms from the governing equation.

In the hybrid-Trefftz finite element context, the general and particular solution functions are coupled in the same basis and used to enforce weakly the domain equations. This strategy offers considerable flexibility over alternative indirect methods since it allows the particular solution functions to contribute to the modelling of the general solution, should the Trefftz-compliant functions be insufficient or the excitation frequency very small. Moreover, the specific choice of the particular solution functions simplifies considerably the calculation of the terms of the finite element solving system, which preserve, in general, the boundary integral expressions that typify Trefftz formulations.

Alternatively, the formulation can be casted using a typical DRM strategy by restricting the role of the particular solution functions to the modelling of the source term. The main advantage of the approach presented here over the DRM is that it is not necessary to integrate the source term approximation functions in order to obtain the particular solution basis. Instead, the approximations of both particular solution and source function are constructed using the same set of trial functions. On the downside, the shape functions that satisfy the corresponding static problem are restricted to regular (or regularly-extended) domains.

Both strategies are applied to a benchmark hyperbolic problem in order to assess their convergence and robustness properties. Their relative advantages and drawbacks are also discussed.

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