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SCALABLE FULLY IMPLICIT SOLVERS FOR EXTENDED MAGNETOHYDRODYNAMICS

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The magnetohydrodynamics (MHD) model is a fluid description of the plasma state. While plasma is made up of independent (but coupled) ion and electron species, the standard MHD description of a plasma only includes ion time and length scales (one-fluid model). Extended MHD (XMHD) includes nonideal effects such electron Hall physics, parallel electron transport, and ion gyroviscosities, which are known to play an important role in certain regimes of interest (notably, hot, moderate-density plasmas, such as those found in the solar corona, the Earth's magnetosphere, and in thermonuclear magnetic-confinement fusion devices). However, numerically, the inclusion of such physical effects is highly nontrivial. Both electron Hall physics and ion gyroviscosities introduce strongly hyperbolic couplings, resulting in dispersive normal modes (waves) with frequencies ω scaling with the square of the wavenumber k^2. In explicit time integration methods, this results in a stringent CFL limit $\Delta t_{CFL} \propto \Delta x^2$, which severely limits their applicability to the study of long-frequency phenomena in XMHD. A fully implicit implementation promises efficiency (by removing the CFL constraint) without sacrificing numerical accuracy as long a dynamical time scales of interest are resolved [1]. However, the nonlinear nature of the XMHD system and the numerical stiffness of its fast waves make this endeavor very difficult. Newton-Krylov methods can meet the challenge provided suitable preconditioning is available.

We propose a successful preconditioning strategy for the 3D primitive-variable XMHD formalism. It is based on "physics-based" ideas [2, 3, 4] in which a hyperbolic system of equations (which is diagonally submissive for $\Delta t > \Delta t_{CFL}$) is "parabolized" to arrive to a diagonally dominant approximation of the original system, which is multigrid-friendly. The use of approximate multigrid (MG) techniques to invert the "parabolized" operator is a crucial step in the effectiveness of the preconditioner and the scalability of the overall algorithm. The parabolization procedure can be properly generalized using the well-known Schur decomposition of a 2×2 block matrix. In the context of XMHD, the resulting Schur complement is a system of PDE's that couples the three plasma velocity components, and needs to be inverted in a coupled manner. Nevertheless, a system MG treatment is still possible since, when properly discretized, the XMHD Schur complement is block diagonally dominant by construction, and block smoothing is effective.

In this presentation, we will discuss the derivation and validity of the physics-based preconditioner for resistive MHD and its generalization to XMHD, the connection with Schur complement analysis, and the system-MG treatment of the associated systems. We will present both linear and nonlinear verification examples, and demonstrate the algorithm using the GEM challenge configuration [5].

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