11th. World Congress on Computational Mechanics (WCCM XI)
5th. European Conference on Computational Mechanics (ECCM V)
6th. European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

NUMERICAL ANALYSIS OF THE DAMPED WAVE EQUATION BY "ENERGETIC" WEAK FORMULATIONS

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Key words: *Wave Equation, Boundary Element Method (BEM), Finite Element Method (FEM), Damping.*

The analysis of damping phenomena that occur in wave propagation is of particular interest, for example, in fluid dynamics, in kinetic theory and semiconductors: the dissipation is generated in the interaction of the waves with the propagation medium and can be also closely related to the dispersion, as in the interactions between water streams and surface waves. For the numerical solution of these problems, we need consistent approximations and accurate simulations even on large time intervals.

For the damped wave equation, we consider extensions of the so-called *energetic* formulation, recently proposed, which was presented and applied to retarded BIEs related to the wave equation without damping terms, directly expressed in the space-time domain with an analysis of important stability properties in time and achieving, with its discrete implementation, significant numerical results [1, 2].

Now, *energetic* BEM and BEM-FEM coupling is applied to problems modeled by 1D damped wave equation, in bounded and unbounded single and multi-domains, and here numerical results will be presented and discussed.

In particular, we will consider a test problem, suggested in [3], which involves the one-dimensional linear wave equation with constant wave propagation velocity c, viscous damping coefficient D and material damping coefficient P for the displacement field $u(x,t)$ in a domain $\Omega = [0, L] \subset \mathbb{R}$, equipped with the following initial and boundary conditions:

$$\left(\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{2D}{c^2}\frac{\partial u}{\partial t} - \frac{P}{c^2}u\right)(x,t) = f(x,t) = 0, \quad x \in \Omega,\ t \in [0,T] \tag{1}$$

$$u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0, \quad x \in \Omega \tag{2}$$

$$u(0,t) = g_D(t) = 0, \quad t \in [0,T] \tag{3}$$

$$\frac{\partial u}{\partial \mathrm{n}}(L,t) = g_N(t), \quad t \in [0,T] \tag{4}$$

In Figure 1, $u(L,t)$ has been computed with *energetic* BEM, fixing $c = 1$, $L = 1$, $D = 0$, varying P and using the discretization parameter $\Delta t = 0.01$.

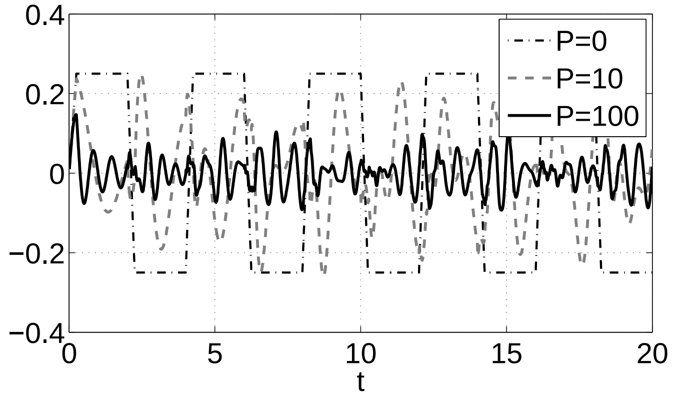

Figure 1: $u(L,t)$, having set $g_N(t) = H[t] - H[t - \frac{1}{4}]$, for $c = 1$, $L = 1$, $D = 0$, varying P.

In order to investigate the diffusive nature of viscous damping in structures a repetitive pulse traction $g_N(t)$, as represented in Figure 2, has been applied. Figure 3 shows $u(L,t)$ computed with *energetic* BEM, fixing $c = 1$, $L = 1$, $P = 0$, varying D and using the discretization parameter $\Delta t = 0.01$.

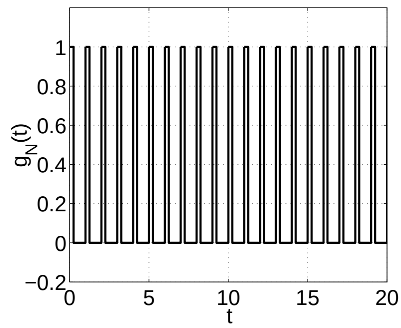

Figure 2: Repetitive pulse traction $g_N(t)$.

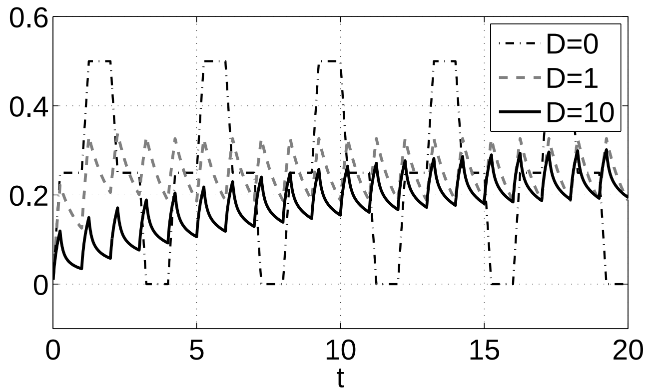

Figure 3: $u(L,t)$, having set $g_N(t)$ as in Figure 2, for $c = 1$, $L = 1$, $P = 0$, varying D.

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