

A REDUCED-ORDER MODELLING APPROACH FOR BRIDGING COMPUTATIONAL AND ANALYTICAL HOMOGENISATION

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Many natural or engineered materials are heterogeneous at a microscopic length scale, but homogeneous at a macroscopic length scale. For a better understanding of the material behaviour, it is necessary to model the macroscale while taking the microscale into account. In homogenisation techniques [2], one defines a representative volume element (RVE) that is small enough to represent a point of the macrostructure, but large enough so that it contains enough heterogeneities of the microscale structure to be statistically representative. The RVE boundary value problem is then solved assuming the strain of the macrostructure is the average of the strain on the RVE, which defines boundary conditions. This returns a stress field over the RVE that is averaged to give an estimation of the corresponding macro-stress. In this contribution, we look at a lattice structure modelled using damage. The non-linearity of the constitutive law and its history-dependency means that an FE^2 [3] strategy is necessary to capture the varying stress-strain relationships at different positions in the material and different timesteps. Since solving the RVE problem for each gauss-point of the related macro-mesh is prohibitively expensive, there is a need for reduced-order modelling. We use the proper orthogonal decomposition (POD) ([1]) to find a reduced space containing all the possible displacements of the RVE. The snapshots are obtained by solving the RVE boundary value problem for various loading paths. To speed-up the computations, system approximation through the discrete empirical interpolation (DEIM) [4] is used and allows the evaluation of the internal forces for only a small subset of the elements making the RVE structure. We show great time savings while preserving the accuracy of the average output stress.

REFERENCES

- [1] L. Sirovich. Turbulence and the dynamics of coherent structures. *Quarterly of applied mathematics*, 561–571, 1987.
- [2] S. Nemat-Nasser and M. Hori. Micromechanics: overall properties of heterogeneous materials, *Elsevier Amsterdam*, volume **2**, 1999.
- [3] F. Feyel. A multilevel finite element (FE2) to describe the response of highly nonlinear structures using generalized continua. *Computer Methods in Applied Mechanics and Engineering*, 192:3233–3244, 2003.
- [4] S. Chaturantabut and D.C. Sorensen. Discrete empirical interpolation for nonlinear model reduction. *Proceedings of the 48th IEEE Conference on Decision and Control*, 4316–4321, 2009.