

## AAR-BASED DECOMPOSITION METHOD FOR LIMIT ANALYSIS

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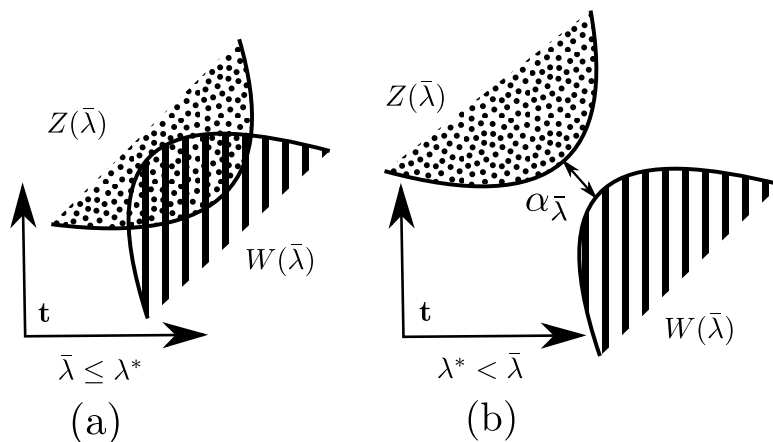
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The analysis of the bearing capacity of structures with a rigid-plastic behaviour can be achieved resorting to computational limit analysis. Recent techniques [2],[3] have allowed scientists and engineers to determine upper and lower bounds of the load factor under which the structure will collapse. Despite the attractiveness of these results, their application to practical examples is still hampered by the size of the resulting optimisation process.

We here propose a method for decomposing a class of convex non-linear programmes which are encountered in limit analysis. These problems have second-order conic memberships constraints and a single complicating variable in the objective function. The method is based on finding the distance between the feasible sets of the decomposed problems, and updating the global optimal value according to the value of this distance. The latter is found by exploiting the method of Averaged Alternating Reflections (AAR) [1], which is here adapted to the optimisation problem at hand. In particular, we re-cast the standard optimisation in limit analysis as the maximisation of the load capacity  $\lambda$  subjected to non-empty intersection of sets, i.e.

$$\begin{aligned}
 \lambda^* = \max_{\mathbf{x}_1, \mathbf{x}_2, \lambda} \lambda & & \lambda^* = \max_{\lambda} \lambda \\
 f_1(\mathbf{x}_1, \lambda) = \mathbf{0} & & Z(\lambda) \cap W(\lambda) \neq \emptyset \\
 f_2(\mathbf{x}_2, \lambda) = \mathbf{0} & \Leftrightarrow & \lambda \in \mathbb{R} \\
 g_1(\mathbf{x}_1) + g_2(\mathbf{x}_2) = \mathbf{0} & & \\
 \mathbf{x}_1 \in K_1 \subseteq \mathbb{R}^{n_1}, \mathbf{x}_2 \in K_2 \subseteq \mathbb{R}^{n_2}, \lambda \in \mathbb{R}. & & 
 \end{aligned}$$

with  $Z(\bar{\lambda})$  and  $W(\bar{\lambda})$  appropriate feasibility sets. Figure illustrates this idea, with  $\lambda^*$  the optimal (maximum) load capacity. The method is specially suited for non-linear problems, and as our numerical results show, its convergence is independent of the number



**Figure 1:** Relation between optimal load capacity  $\lambda^*$  and intersection of sets  $Z(\bar{\lambda}) \cap W(\bar{\lambda})$ .

of variables of each sub-domain. We have tested the method with problems that have more than 10000 variables.

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