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IMPLEMENTATION OF MATERIAL MODELING APPROACHES AT FINITE STRAINS USING A HIGHLY ACCURATE NUMERICAL DERIVATIVE SCHEME

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Numerical simulation of the nonlinear mechanical behavior of materials undergoing finite strains remains an important and challenging topic in computational mechanics, in particular with view to micro-heterogeneous materials. For their finite element implementation, the stresses and consistent tangent moduli need to be explicitly specified from the strain energy function for hyperelastic models or from the incremental stress potential for standard dissipative materials, see e.g., [1]. In most cases they can be obtained mainly in terms of the first and second derivative of the potential function with respect to the change of total deformation. The exact calculation of the stresses determines accuracy of the numerical simulation, and the consistent algorithmic tangent moduli are required to achieve quadratic convergence in an iterative solution scheme as well as to detect material instabilities in localization analysis, cf. e.g., [2]. However, for some material models their analytic derivation may be elaborate to be calculated or implemented due to its mathematical complexity, in particular for anisotropic media. In those cases the approaches using numerical differentiation may be a useful alternative reducing the implementation time in particular for scientific development purposes, see e.g., [3], [4], or [5]. However, most of them make use of only first order numerical derivatives, where they compute the approximated tangent moduli from the stresses. Although the often-used classical finite difference (FD) scheme can be applied consecutively, this will end up in a poorly accurate scheme being sensitive with respect to perturbation values, especially for the second derivative. An alternative based on the complex-step derivative approximation (CSDA), cf. [6], is also problematic since no second derivatives can be computed. To the best of our knowledge, a robust numerical scheme for the first and second derivatives in order to derive at once the stresses and consistent tangent moduli directly from the strain energy function has not been reported in the literature. Fike [7] developed the exact first and second derivative calculation independent on the choice of perturbation values using hyper dual numbers. Therefore, here we make use of hyper dual numbers and propose a numerical scheme for the calculation of stresses and tangent moduli which are almost identical to the analytic ones. Its uncomplicated implementation and accuracy is illustrated by some representative numerical examples.

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