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LOW-FREQUENCY SHAPE FUNCTIONS ON THE LOGARITHMIC SPACE

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To obtain good convergence characteristics in multigrid applications, the coarse grid algorithm must operate efficiently on the low-frequency part of the approximation error. At the same time, its influence on the high frequencies must be minimized.

In order to realize these objectives, we propose a finite element model that focuses on the approximation of the low-frequency part of a deformation, using a small number of degrees of freedom.

In contrast to the standard Ritz-Galerkin approach, the (internal and external) degrees of freedom are given as coefficients of shape functions on a Lie algebra, i.e. on the logarithmic space, allowing to reduce the number of finite elements and total degrees of freedom without incurring the locking phenomena associated with linear shape functions. Choosing appropriate basis vectors on the Lie algebra and suitable shape functions is of crucial importance for the performance of the model.

In the case of a two-dimensional Bernoulli beam, the proposed model employs shape functions defined on the Lie algebra $\mathbb{C} \rtimes_{\mathrm{id}} \mathfrak{gl}(1, \mathbb{C})$. These shape functions induce a tight coupling of translations, rotations and dilatations within a single finite element. Given the deformation (1), the similarity transformations $\mathbf{S}(\xi) \in \mathfrak{gl}(1, \mathbb{C})$ and the translations $\mathbf{u}(\xi) \in \mathbb{C}$ related to a neighborhood of a point $\mathbf{x}(\xi)$ on the neutral axis jointly depend on a small set of degrees of freedom.

$$\begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} \mapsto \exp \begin{pmatrix} \mathbf{S} & \mathbf{u} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = \begin{pmatrix} \mathrm{e}^{\mathbf{S}} & \mathbf{S}^{-1}\left(\mathrm{e}^{\mathbf{S}} - I\right)\mathbf{u} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} \tag{1}$$

In the limit $\mathbf{S} \to 0$, implying $\mathrm{e}^{\mathbf{S}} \to I$ and $\mathbf{S}^{-1}\left(\mathrm{e}^{\mathbf{S}} - I\right) \to I$, the model collapses to the standard finite element approach.

For different load conditions, we obtain approximations based on a limited number of up to eight parameters that closely match the solutions resulting from standard finite element models employing a much larger number of degrees of freedom. We also explore the three-dimensional case and applications of the method for shell models.