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PERIODIC MOTIONS OF COUPLED IMPACT OSCILLATORS

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In this work, we study the existence and stability of time-periodic oscillations in a chain of linearly coupled impact oscillators reminiscent of a model analyzed in [2], for rigid impacts without energy dissipation. We introduce a numerical method allowing to compute branches of time-periodic solutions *when an arbitrary number of nodes undergo rigid impacts.* For this purpose, we reformulate the search of periodic solutions as a boundary value problem incorporating unilateral constraints. We illustrate this numerical approach by computing some families of nonlinear spatially localized modes (breathers) and extended modes.

The dynamics is described by the following complementarity system

$$\ddot{y}_n + y_n - \gamma\,(\Delta y)_n = \lambda_n, \quad n \in \mathbb{Z}, \tag{1}$$

$$0 \le \lambda \perp (y + \mathbb{1}) \ge 0, \tag{2}$$

$$\text{if } \dot{y}_n(t^-) < 0 \text{ and } y_n(t) = -1 \text{ then } \dot{y}_n(t^+) = -\dot{y}_n(t^-), \tag{3}$$

where $(\Delta y)_n = y_{n+1} - 2\,y_n + y_{n-1}$ defines a discrete Laplacian operator, $\mathbb{1}$ denotes the constant sequence with all terms equal to unity and $\gamma \ge 0$ is a parameter. Non-dissipative impacts occur for $y(t) = -1$ and give rise to impulsive reaction forces $\lambda(t)$. We look for T-periodic solutions even in time, and assume each particle undergoes at most one impact during each period of oscillation. Introducing the splitting $y = (y^{(0)}, y^{(1)}, y^{(2)})$ corresponding to $\mathbb{Z} = I_0 \cup I_1 \cup I_2$, the above system can be reformulated as a boundary value problem on a half-period interval $(0, T/2)$,

$$\ddot{y}_n + y_n - \gamma\,(\Delta y)_n = 0, \quad n \in \mathbb{Z}, \quad t \in (0, T/2), \tag{4}$$

with boundary conditions

$$\dot{y}^{(i)}(0) = 0 \text{ for } i \in I_0 \cup I_1, \quad y^{(2)}(0) = -\mathbb{1}, \quad \dot{y}^{(i)}(T/2) = 0 \text{ for } i \in I_0 \cup I_2, \quad y^{(1)}(T/2) = -\mathbb{1}, \tag{5}$$

and constraint

$$y(t) + \mathbb{1} > 0, \quad t \in (0, T/2). \tag{6}$$

We solve this problem numerically for a chain of $N = 100$ oscillators with periodic boundary conditions. We use a shooting method, i.e. determine $z = (y^{(0)}(0), y^{(1)}(0), \dot{y}^{(2)}(0)) \in \mathbb{R}^N$ such that the three boundary conditions of (5) at $t = T/2$ are satisfied. This requires to solve a linear system for z obtained through time-integration of the linear ODE (4) (the case of nonlinear local

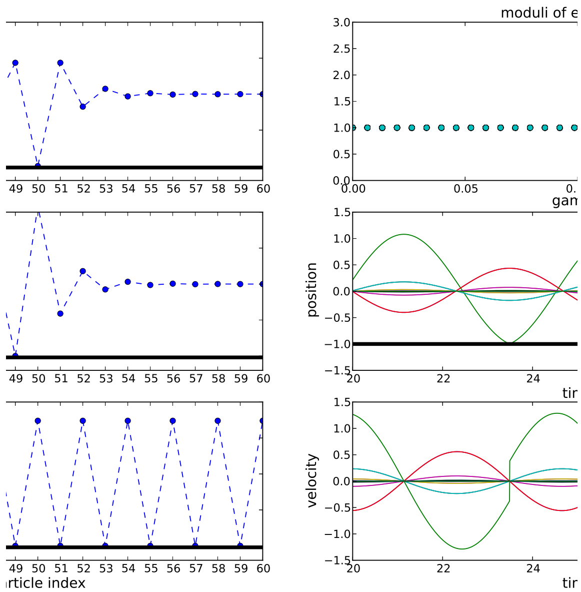

Figure 1: Computation of different periodic solutions for $T \approx 4.7$. The left column displays particle positions at $t = 0$ for $\gamma = 0.16$, for two breather solutions with $I_2 = \{50\}, I_1 = \emptyset$ (site-centered breather, top panel) and $I_2 = \{49\}, I_1 = \{50\}$ (bond-centered breather, middle panel), and for a nonlinear normal mode with spatial period two and $I_0 = 2\mathbb{Z}, I_1 = \emptyset$ (bottom panel). These solutions can be continued for $\gamma \in [0, \gamma_{\max})$ with $\gamma_{\max} \approx 0.19$. The site-centered breather is linearly stable for $\gamma < \gamma_c \approx 0.13$, after which it becomes unstable (the top right panel displays the moduli of the corresponding Floquet eigenvalues). The time evolution of the position and velocity of the impacting particle ($n = 50$) is illustrated over a few periods for $\gamma = 0.16$ (right column, middle and bottom panels). The bond-centered breather and nonlinear normal mode are both unstable for all values of γ.

or interaction potentials could be addressed similarly using a Newton method). The constraint (6) is checked *a posteriori*. Solution branches are continued for fixed values of T, varying the linear stiffness γ and starting from the uncoupled (or "anticontinuum") limit $\gamma = 0$ [1]. In this limit, for all fixed $T \in (\pi, 2\pi)$, a choice of impacting particles and phases (determined by I_1, I_2) selects a unique solution which can be continued up to some maximal value of γ. The linear stability of periodic solutions is analyzed through the eigenvalues of an associated monodromy matrix. To perform this computation, we integrate (1)-(2)-(3) numerically using the Siconos software for nonsmooth dynamical systems [3]. As an example, we describe in Figure 1 three families of periodic solutions obtained with this method, namely two breather solutions (site-centered or bond-centered) and a spatially extended solution (nonlinear normal mode). The computation of periodic solutions based on the above approach is much more effective than numerical continuation of periodic solutions based on compliant models. In the latter case, impacts are described by smooth nonlinear Hertzian type potentials leading to stiff ODE and costly numerical continuation. Future extensions of this work will include an analytical continuation and stability analysis based on the same approach, the inclusion of dissipative impacts and forcing, and the application of the method to more complex finite-element models of continuous systems under impacts.

REFERENCES

[1] S. Flach and A. Gorbach. Discrete breathers : advances in theory and applications, *Physics Reports 467* (2008), 1-116.

[2] O.V. Gendelman and L.I. Manevitch, Discrete breathers in vibroimpact chains: analytic solutions, *Phys. Rev. E 78* (2008), 026609.

[3] V. Acary and F. Pérignon. An introduction to Siconos. Technical Report, 2007, pp. 45. RT-0340. http://siconos.gforge.inria.fr/