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NUMERICAL APPROACH FOR VISUALIZATION OF THE BUCKLING SPHERE BY MEANS OF RESOLVING THE CONSISTENTLY LINEARIZED EIGENPROBLEM

S. Pavlicek*, X. Jia and H. A. Mang

Institute for Mechanics of Materials and Structures, Vienna University of Technology,
Karlsplatz 13, 1120 Vienna, Austria
stefan.pavlicek@tuwien.ac.at

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Recently, a concept of energy-based categorization of buckling was proposed [1]. It represents a symbiosis of mechanics of solids and spherical geometry. The geometrical basis is the so-called buckling sphere [1].

The vertex of a vector **a** of unit length moves along the surface of an octant of this sphere. The movement of this vector is controlled by a dimensionless load parameter λ. It describes a surface curve on the buckling sphere, an arbitrary point of which is defined by the azimuth angle $\varphi(\lambda)$ and the zenith angle $\theta(\lambda)$. The spherical coordinates are related to ratios of energetical quantities, such as the ratios of the buckling energy to the total strain energy.

The mathematical tool of the initially mentioned concept is the so-called conistently linearized eigenproblem [2]. Its formulation reads as

$$\boldsymbol{A}_j(\lambda) \cdot \boldsymbol{v}_j^*(\lambda) = \boldsymbol{0}\,, \quad j = 1, 2, ..., N, \tag{1}$$

with

$$\boldsymbol{A}_j := \widetilde{\boldsymbol{K}}_T + (\lambda_j^* - \lambda) \cdot \dot{\widetilde{\boldsymbol{K}}}_T\ , \tag{2}$$

where $\widetilde{\boldsymbol{K}}_T$ represents the stiffness matrix in the frame of Finite Element Method and $(\lambda_j^* - \lambda,\, \boldsymbol{v}_j^*)$ is the j-th eigenpair.

The global stiffness Matrix $\widetilde{\boldsymbol{K}}_T$ is computed by means of the finite element software MSC.MARC.

Computation of φ and θ requires solution of the CLE for the first eigenpair which is associated with loss of stability. A necessary requirement for this solution is a sufficiently accurate approximation of the first derivative of $\widetilde{\boldsymbol{K}}_T$ with respect to λ. For determination of θ also a sufficiently accurate approximation of the second derivative of $\widetilde{\boldsymbol{K}}_T$ with respect to λ is needed.

The mentioned derivatives of the stiffness matrix are expressed as forward difference expressions. Since the source code of MARC is not accessible, the matrix $\boldsymbol{K}_T(\boldsymbol{q}(\lambda^{(0)}) + h\dot{\boldsymbol{q}}(\lambda^{(0)}))$, which is the basis for the finite difference expression, is evaluated with the help of a built-in Newton-Raphson scheme, where h represents a small positive number and $\dot{\boldsymbol{q}}$ denotes the derivative of the nodal displacement vector $\boldsymbol{q}$ with respect to λ.

The strategy devised for the indicated implementation was to modify the full Newton-Raphson algorithm as part of the equilibrium iteration after application of a load increment, such that only the required finite difference expression is computed. This approach was also used in the context of the arc-length method, which is advantageous in case of loss of stability by snap-through. As a remedy for numerical problems related to the limited precision of the computing mechanics, which becomes relevant as h tends to zero, the eigenvalue $\lambda_1^* - \lambda$, as well as the quadratic forms $\boldsymbol{v}_1^* \cdot \widetilde{\boldsymbol{K}}_T \cdot \boldsymbol{v}_1^*$ and $\boldsymbol{v}_1^* \cdot \dot{\widetilde{\boldsymbol{K}}}_T \cdot \boldsymbol{v}_1^*$ are approximated by evaluating them at different fractions of $h/\Delta\lambda$, where $\Delta\lambda$ is the considered load step, and using the results for regression analysis.

Determination of θ, following computation of φ, requires the solution of the following linear eigenproblem:

$$\left[\frac{\alpha_1}{\lambda_1^* - \lambda}\dot{\widetilde{\boldsymbol{K}}}_T + \ddot{\widetilde{\boldsymbol{K}}}_T\right] \cdot \hat{\boldsymbol{v}}_1 = \mathbf{0}\,. \tag{3}$$

The strategy for this solution is analogous to the one for the solution of (1).

REFERENCES

[1] Categorization of Buckling by Means of Spherical Geometry, 2013, H. A. Mang, X. Jia, San Diego, Advances in Computational Mechanics (ACM).

[2] P. Helnwein. Zur initialen Abschaetzbarkeit von Stabilitaetsgrenzen auf nichtlinearen Last-Verschiebungspfaden elastischer Strukturen mittels der Methode der Finiten Elemente [in German; On *ab initio* assessability of stability limits on nonlinear load-displacement paths of elastic structures by means of the finite element method], Vienna University of Technology, 1997