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MULTISCALE METHOD USING STATIC AND TRANSIENT SUBSCALES TO SOLVE TRANSPORT FLOW PROBLEMS

L. Catabriga[1], A.M.P Valli[1], R.C. Almeida[2] and I.P. Santos[3]

[1] LCAD/DI/UFES, Vitória/ES, luciac@inf.ufes.br, avalli@inf.ufes.br
[2] LNCC, Rio de Janeiro/RJ, rcca@lncc.br
[3] UFES, São Mateus/ES, isaacps@gmail.com

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The numerical solution of transport equations and inviscid compressible flows using the Galerkin formulation may exhibit global spurious oscillations. More accurate and stable results can be obtained using stabilized or multiscale methods. The multiscale approach consists of decomposing the approximation space into resolved (coarse) scales and unresolved (subgrid) scales such that the weak form of the problem is split into two subproblems: one for the coarse scales and one for the subgrid scales. From this point of view, the attempts to recover stability of the resolved scale solution may be viewed as a way of improving the simulation by considering the effects of the smallest scales on the larger ones [3]. The Nonlinear Subgrid Stabilization (NSGS) method is a step towards the development of a two-scale method whose stability and convergence properties do not rely on tune-up parameters [5, 6].

The NSGS method was analyzed in [6] to advection-diffusion-reaction problems and was proved to be stable and to yield optimal convergence rates by assuming that the grid is quasi-uniform, but presents the drawback of requiring the solution of a system twice as large as that associated with the resolved scale resolution. Motivated by eddy viscosity models in which the dissipation mechanism is introduced either on all scales or on the subgrid scale, Arruda et al. [2] proposed a discontinuous two scale method where an artificial diffusion appears on all scales, named Dynamic Diffusion method (DD). This method results in a free parameter method in which an artificial diffusion model acts isotropically and is locally adapted to guarantee stability of the resolved scale solution. The stability provided by the DD method outperforms those provided by some subgrid diffusion approaches for transport problems and some discontinuous capturing methods. In particular, it reformulates, using broken spaces, the Consistent Approximate Upwind Petrov-Galerkin (CAU) finite element model [1].

In this work we present a finite element formulation for transient advection-diffusion and inviscid compressible flow problems using the DD method with static and transient subscales. In order to reduce the computational cost typical of two-scale methods, the subgrid scale space is defined using bubble functions whose degrees of freedom are locally eliminated in favor of the degrees of freedom that live on the resolved scales. The resulting nonlinear system of ordinary differential equations are discretized in time using finite difference schemes and the linear algebraic systems are solved by a nodal block-diagonal preconditioned GMRES method. Moreover, the final discrete setting is implemented and evaluated into a local data structure framework by using the well known element-by-element storage scheme. Performance and accuracy comparisons are based on benchmark 2D problems.

Numerical experiments were conducted to illustrate the behavior of the transient DD formulation applied to advection-diffusion-reaction equations and compressible Euler equations using an implicit predictor/multicorrector scheme [4]. For advection-diffusion-reaction problems, the comparison with the results obtained using the SUPG/CAU method indicates that the DD method performed better, both in terms of number of GMRES iterations and the computational time spent in solving the problem. For the Euler equations, a set of classic experiments shown that the DD method with transient subgrid scales result in more accurate solutions than the stabilized methods SUPG/CAU e SUPG/YZβ. However, for the classical experiments of oblique and reflective shocks we observed a conditionally stable behavior. Therefore we are developing different time integration schemes in order to obtain unconditionally stable methods.

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