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COUPLING OF DISCONTINUOUS GALERKIN AND FINITE DIFFERENCES METHODS FOR SIMULATION SEISMIC WAVES

Julien Diaz[1], Vadim Lisitsa[2], Vladimir Tcheverda[3] and Dmitry Vishnevsky[4]

[1] Inria Bordeaux Sud-Ouest, Equipe-Projet Magique-3D IPRA-LMA, Universite de Pau et des Pays de l'Adour, BP 1155, 64013 Pau Cedex, julien.diaz@inria.fr
[2] Institute of Petroleum Geology and Geophysics SB RAS, 3 Koptug pr., Novosibirsk, Russia, 630090, lisitsavv@ipgg.sbras.ru
[3] Institute of Petroleum Geology and Geophysics SB RAS, 3 Koptug pr., Novosibirsk, Russia, 630090, cheverdava@ipgg.sbras.ru
[4] Institute of Petroleum Geology and Geophysics SB RAS, 3 Koptug pr., Novosibirsk, Russia, 630090, vishnevskydm@ipgg.sbras.ru

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Nowadays, finite differences, due to their simplicity, universality and high computational efficiency, are the most common approach to simulate wave propagation in complex large-scale models [1]. Accuracy of the finite-difference simulation is typically acceptable for a wide range of real-life applications in seismic prospecting. However, presence of highly contrast interfaces such as free-surface and sea-bottom with complex topography significantly reduces quality of the simulations. On the other hand implementation of finite elements on unstructured triangular (tetrahedral) meshes increases the smoothness of the interface approximation thus significantly improves the accuracy of the synthetic wavefields. However, mesh generation is a complex task and computational intensity of the finite element algorithms is higher than that of finite differences, even for the same degrees of freedom used by both methods.

In this paper, we present an original algorithm where the standard staggered grid scheme (SSGS) [2] is combined with the discontinuous Galerkin (DG) method. The DG method is applied only in the near-surface part of the model, in order to match the wave interaction with sharp interfaces and the SSGS is used in the main part of the model, in order to reduce computational intensity of the algorithm. Here, we focus on the velocity-stress formulation of the elastic wave equation and we consider an approximation by second order finite differences and up to P3 element for the DG method. In order to couple DG on the triangular grid with FD, we suggest to introduce a transition zone with DG-P0 elements on a regular rectangular grid. As it follows from [3] P0 DG on the rectangular

grid coincide with conventional; i.e. non-staggered, finite-difference scheme. Thus the algorithm can be divided into two independent parts:

- coupling of DG on the triangular grid with DG on the rectangular grid;
- coupling of the conventional finite-difference scheme with the standard staggered grid scheme.

The first part of the coupling can be applied in a straightforward manner as it is a case of hp-adaptivity, which is a natural part of the DG based algorithms. The second part of the approach is based on a specific property of the conventional finite-difference scheme for the velocity-stress formulation of elastic wave equation, which induces artificial "plus-minus" modes, while SSGS does not allow such modes. So, the formulae to couple the two schemes are designed in such a way that the true solution propagates through the interface between the two grids but the artifacts are completely reflected. Similar principles are used to couple Lebedev scheme with SSGS in [4]. As the result, the artificial reflections (the main numerical error) caused by the coupling of DG and standard staggered grid scheme are low as $10^{-3} - 10^{-4}$ of the incident wave, which is an acceptable level for the seismic prospecting.

The presented algorithm allows one for accurately simulating wave propagation in presence of a complex near-surface part of the model and surface topography, but preserve computational efficiency of finite-difference methods.

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