11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

FAST MULTIPOLE BOUNDARY ELEMENTS METHOD FOR MULTIZONE PROBLEMS

T. Trinh[1*], S. Mouhoubi[1], C. Chazallon[1] and M. Bonnet[2]

[1] Laboratory ICUBE, UMR 7357 - INSA of Strasbourg, 24 bd de la Victoire, 67084 Strasbourg
quoctuan.trinh@insa-strasbourg.fr / saida.mouhoubi@insa-strasbourg.fr
[2] Equipe POEMS - ENSTA ParisTech, 1024 boulevard des Marchaux, 91762 Palaiseau Cedex

Key words: *Elastostatics, Fracture, SGBEM, FMM, multizone, Flexible GMRES.*

Domains containing internal boundaries such as interfaces between materials occur in many applications: composite materials, geophysical simulations... The treatment of these situations requires the continuity condition across the interfaces. In a generic multizone system, there are usually additional unknowns due to the presence of interfaces. These unknowns belong to all adjacent bodies and are governed by the continuity condition. Therefore, the construction of the global coefficient matrix and the resolution of the system become more difficult and complicated. Among many other alternatives, the Boundary Element Method (BEM) [1] has been regarded as a powerful and efficient tool in treating multizone and fracture analysis because of its distinct advantages. Furthermore, by coupling the BEM with the Fast Multipole Method (FMM) [4, 5, 6], all the limitations of a normal BE analysis are greatly reduced which renders the solution of such problems computationally fast and efficient.

In our work, the symmetric Galerkin approach (SGBEM) and the technique described in [2, 3] has been employed to construct the symmetric matrix system of multizone problems. We also introduce the application of the Fast Multipole algorithm for the multizone SGBEM in the context of elastostatic and fracture mechanics. The numerical results yield a very good agreement with the references (Fig.1 and Fig.2). With help of the FMM, the constructed boundary elements algorithm is also able to solve large-scale problems ($N \simeq 10^6$) with modest computational resources. The study leads to the treatment of some complex but promising realistic problems where the crack propagation is taken into account.

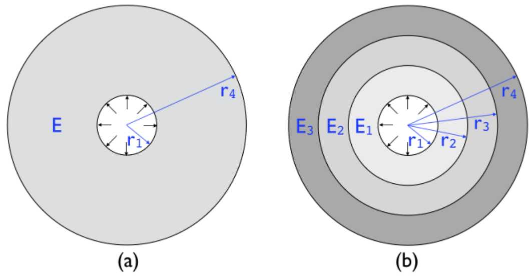

Figure 1: Spherical Envelope under internal pressure (a) 1-layer body $E = 1, \nu = 0.3$ (b) 3-layer body ($r_1 = 1, r_2 = 2, r_3 = 3, r_4 = 4$) of identical material properties: $E_1 = E_2 = E_3 = 1, \nu_1 = \nu_2 = \nu_3 = 0.3$

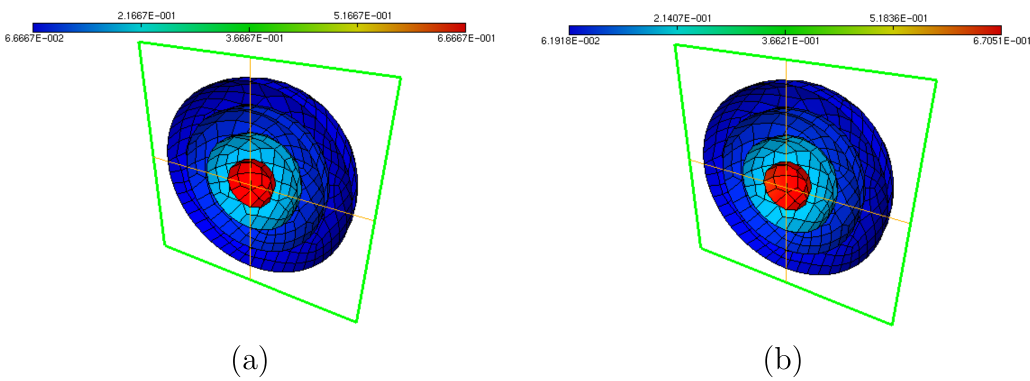

Figure 2: Visualization of the radial displacement: (a) exact results (b) numerical results of FM-SGBEM (relative error $< 1\%$)

REFERENCES

[1] M. Bonnet. Equations integrales et elements de frontiere. Application en mecanique des solides et des fluides. *CNRS Edition*, 1995.

[2] L. J. Gray and H. Paulino. Symmetric Galerkin boundary integral formulation for interface and multizone problems. *Int. J. Numer. Meth. Engng.*, vol. 40::pp 3085-3101, 1995.

[3] M. Magonari. Boundary element techniques for three dimensional problems in Elastostatics, *PhD thesis*, 2004

[4] V. Rokhlin, Rapid solution of integral equations of classical potential theory. *J. Comp. Phys., vol.60.pp. 187-207*, 1985.

[5] K. I. Yoshida, Application of fast multipole to boundary integral equation method. *PhD thesis*, 2001.

[6] S. Chaillat, Methode multipole rapide pour les equations integrales de frontieres en elastodynamique 3D. Application a la propagation d'ondes sismiques. *Phd thesis*, 2008.