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PARALLEL ADAPTIVE-MULTILEVEL BDDC

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The *Balancing Domain Decomposition by Constraints* (BDDC) was developed in [1] as a primal alternative to the *Finite Element Tearing and Interconnecting - Dual, Primal* (FETI-DP) [2]. Both methods aim at parallel solution of large-scale systems of linear equations arising in finite element analysis. Unlike earlier domain decomposition methods, they introduce constraints to impose equality of new 'coarse' variables on substructure interfaces, such as values at substructure corners or weighted averages over edges and faces. These constraints give rise to a global coarse problem.

An adaptive enrichment for BDDC and FETI-DP was proposed in [3], with the added coarse functions built from eigenproblems based on adjacent pairs of substructures. Such approach allows to generate new constraints such that an idicator of the resulting condition number of the preconditioned operator is bellow a prescribed value. The *Adaptive BDDC* allows to solve numerically difficult problems such as those with variable material coefficients, rough subdomain interfaces, or bad-shaped elements, which may fall beyond the capabilities of standard BDDC with conventional constraints.

Another bottleneck of the original BDDC method is solving the coarse problem in the case of many subdomains. Since the coarse problem in BDDC resembles the structure of the finite element problem, it is straightforward to apply the method recursively to solve the coarse problem only approximately, as suggested already in [1] and described in detail in the form of a multilevel method in [4]. While reducing the cost of solving the coarse problem, the condition number bound of *Multilevel BDDC* deteriorates exponentially with increasing number of levels, and the method can require many iterations.

We combine the adaptive and multilevel approaches to the BDDC method in order to develop its variant that would preserve parallel scalability with an increasing number of subdomains and also retain good convergence properties. The adaptive method is applied on each decomposition level. The developed open-source parallel implementation of the *Adaptive-Multilevel BDDC* method shows good scalability to tens of thousands of cores as well as applicability to very large and difficult problems. The presentation is based on [5].

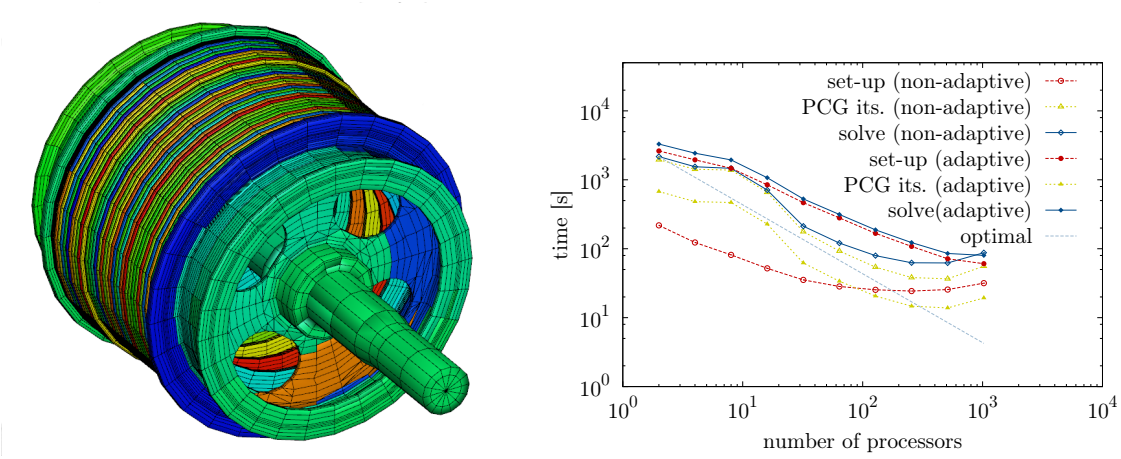

Figure 1: Linear elasticity analysis of a mining reel; (a) Finite element discretization resulting in 1.7M degrees of freedom and substructuring into 1024 subdomains. Data by courtesy of Jan Leština and Jaroslav Novotný. (b) Computational time separately for set-up and PCG phases, and their sum ('solve'), comparison of *non-adaptive* (610 its.) and *adaptive* (200 its.) *parallel 2-level BDDC*.

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