11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

A DISSIPATION-BASED STATE UPDATE ALGORITHM FOR ISOTROPIC ELASTO-PLASTIC HARDENING MATERIALS

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Key words: *Plasticity, Hardening, Incremental problems, Energy minimization, Dissipation, Integration algorithm*

The solution of inelastic structural boundary value problems in a typical finite element implementation requires the numerical integration of the material constitutive law at each Gauss point. In the context of hardening elasto-plastic media, the kinematic variables needed to describe the material state are the total strain $\boldsymbol{\varepsilon}$, additively decomposed into its elastic $\boldsymbol{\varepsilon}^{\mathrm{e}}$ and plastic $\boldsymbol{\varepsilon}^{\mathrm{p}}$ parts, and the strain-like internal variables $\boldsymbol{\alpha}$, i.e. kinematic and isotropic hardening variables. The corresponding conjugated stress-like variables are the stress $\boldsymbol{\sigma}$ and the stress-like internal variables $\boldsymbol{q}$ respectively.

In a strain-driven framework, the material state has to be updated for a given total strain increment. A standard approach for the integration of plastic evolution equations is the use of elastic-predictor inelastic-corrector return map algorithms. In a first step stress and stress-like hardening variables are updated with the assumption that no plastic evolution takes place. In case the elastic trial state is not admissible, a solution for plastic evolution equations, i.e. flow rule, hardening law and plastic consistent condition, is sought for in the plastic corrector step. To this purpose, a common strategy in practical application is a backward-Euler-type approximation of the governing equations, leading to the so-called closest-point projection algorithm (e.g. see [1]). Under the assumptions of (i) associative flow rule, (ii) convex Helmholtz free energy function $\psi(\boldsymbol{\varepsilon}^{\mathrm{e}}, \boldsymbol{\alpha})$ and (iii) convex yield function $f(\boldsymbol{\sigma}, \boldsymbol{q})$, the approximated equations governing the plastic evolution can be shown to be the first-order necessary conditions of a unilaterally constrained variational problem. In that approach, a functional involving the complementary energy $\chi(\boldsymbol{\sigma}, \boldsymbol{q})$, i.e. the Fenchel conjugate of the free energy, has to be minimized with respect to stress $\boldsymbol{\sigma}$ and stress-like hardening variables $\boldsymbol{q}$ under the constraint of plastic admissibility $f(\boldsymbol{\sigma}, \boldsymbol{q}) \leq 0$ [2]. However the constrained character of the return map strategy and the numerical difficulties in convergence when the yield surface presents singularities or points with large curvature motivate the development of alternative techniques.

The present paper focuses on an equivalent statement of the aforementioned variational formulation (e.g. see [3]). In a time-discrete framework, this allows to update the material state from time t_n to time t_{n+1} by solving the infimum problem

$$\inf_{\{\Delta\boldsymbol{\varepsilon}^{\mathrm{p}},\Delta\boldsymbol{\alpha}\}}\{\psi(\boldsymbol{\varepsilon}_{n+1}-\boldsymbol{\varepsilon}_n^{\mathrm{p}}-\Delta\boldsymbol{\varepsilon}^{\mathrm{p}},\Delta\boldsymbol{\alpha})+D(\Delta\boldsymbol{\varepsilon}^{\mathrm{p}},\Delta\boldsymbol{\alpha})\}, \tag{1}$$

where $\Delta\boldsymbol{\varepsilon}^{\mathrm{p}}=\boldsymbol{\varepsilon}_{n+1}^{\mathrm{p}}-\boldsymbol{\varepsilon}_n^{\mathrm{p}}$, $\Delta\boldsymbol{\alpha}=\boldsymbol{\alpha}_{n+1}-\boldsymbol{\alpha}_n$ are the increments of plastic strain and strain-like internal variables respectively and

$$D(\Delta\boldsymbol{\varepsilon}^{\mathrm{p}},\Delta\boldsymbol{\alpha})=\sup_{\substack{\{\boldsymbol{\sigma},\boldsymbol{q}\}\\ f(\boldsymbol{\sigma},\boldsymbol{q})\le 0}}\{\boldsymbol{\sigma}\cdot\Delta\boldsymbol{\varepsilon}^{\mathrm{p}}+\boldsymbol{q}\cdot\Delta\boldsymbol{\alpha}\} \tag{2}$$

is the dissipation function, i.e. the support function of the elastic domain.

The present work exploits this dissipation-based variational formulation to perform the material state update. To this purpose, a two-step algorithm is proposed. In the first step, an elastic prediction of the updated material state is carried out. In case it is not plastically admissible, the infimum problem (1) is solved adopting the Newton-Raphson method. An efficient strategy to compute the dissipation function for isotropic yield criteria is proposed. In particular, adopting the Haigh-Westergaard representation (e.g. see [4]) for the analysis of isotropic yield functions, the supremum problem (2) is reduced to a non-linear scalar equation. Moreover closed-form expressions for the gradient and the Hessian of Haigh-Westergaard coordinates and dissipation function are presented. With the aim of proving the robustness and stability of the proposed algorithm, numerical tests on a single integration point and FEM simulations are provided. This numerical approach appears to be competitive with the typical return map strategy, especially when the yield surface presents singularities or points with large curvature because no difficulty in convergence arises.

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