

## NON-ASSOCIATIVE LIMIT ANALYSIS USING DISCONTINUITY LAYOUT OPTIMIZATION

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Using the associative flow rule, limit analysis can be used to directly estimate the collapse load for a given problem. However, use of the associative flow rule requires that the yield function and the plastic potential of the material are coincident, which can overestimate the amount of dilation which will occur, e.g. in the case of naturally occurring frictional soils, such as sands and gravels. It can therefore result in non-conservative predictions when applied to certain problems, such as those where there is a high degree of confinement.

Discontinuity layout optimization (DLO) has successfully been used to obtain upper bound limit analysis solutions using the associative flow rule [1]. DLO can also be applied to problems involving non-associative friction using an iterative algorithm originally developed for assemblies of rigid blocks [2], but extended for use with DLO by Babiker et al. [3]. In the algorithm, a sequence of linear programs are solved, using the associative solution as a starting point. At each iteration a given discontinuity is modelled with a cohesive-frictional material with properties designed to give an equivalent shear resistance, as indicated in Figure 1. In effect this maintains compliance with the yield function on each discontinuity but allows modification of the plastic potential so that lower or zero dilation can be modelled. This algorithm is applied here to complex soil-structure interactions. For this purpose, a special version of the LimitState:GEO software [4] has been developed. Example mechanisms for associative and non-associative analyses of anchor pullout models are shown in Figure 2, clearly indicating the change in predicted mechanism. In this case the non-associative pullout load was approximately 20 % lower. Note that, unlike associative solutions, non-associative solutions do not necessarily have a clear upper or lower bound status [5], though it is possible to estimate the range of possible non-associated collapse loads corresponding to a specific mechanism.

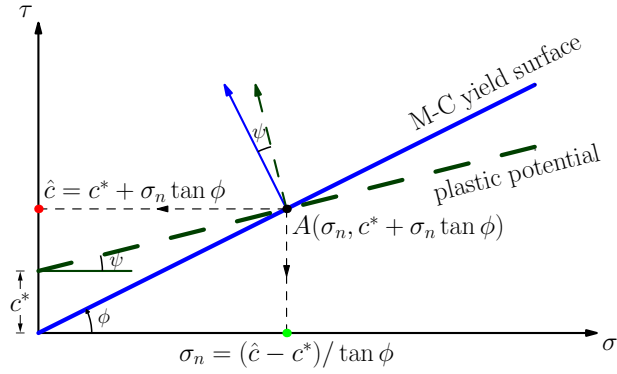


Figure 1: Converged iterative solution: stress state satisfying plastic and yield surface, after [6]

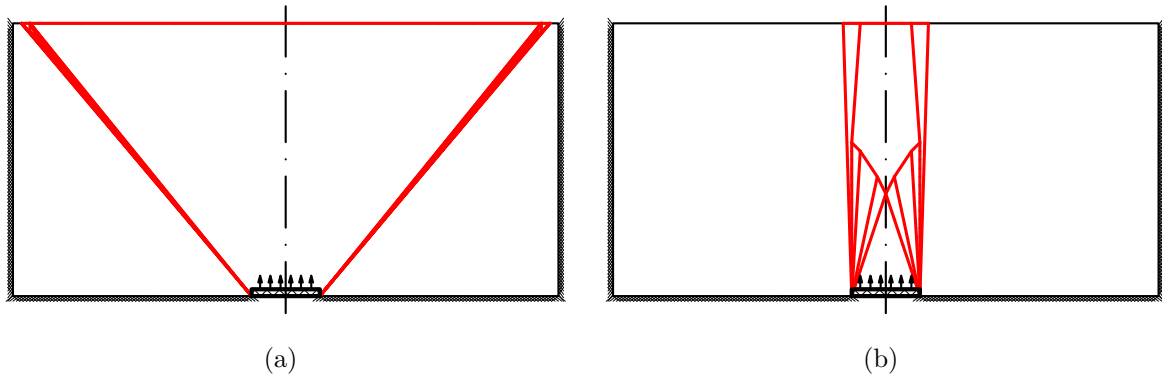


Figure 2: Anchor Problem in sand, with  $\phi = 40^\circ$  (after [6]): (a) associative failure mechanism ( $\psi = 40^\circ$ ); (b) non-associative failure mechanism ( $\psi = 0^\circ$ )

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