ISOGEOMETRIC KIRCHHOFF-LOVE SHELL FORMULATION FOR BIOLOGICAL MEMBRANES

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Biological membranes are present in many clinical applications and their modeling is becoming increasingly important to better understand disease and design more effective medical devices. Remarkable examples of biological membranes are skin, alveoli, blood vessels and heart valves. We can model thin membranes with the Kirchhoff-Love kinematics for which the parametrization of the surface requires at least $C^1$ continuity. NURBS-based isogeometric analysis offers a natural solution to these geometric requirements as they can easily guarantee continuity of $C^1$ or higher over the entire domain [1]. Despite the rising popularity of isogeometric shell descriptions, there is lack of emphasis on arbitrary material models, especially anisotropic constitutive laws, which are of primary importance in biomechanics modeling.

Here we present a Kirchhoff-Love shell formulation parametrized in terms of curvilinear convective coordinates using NURBS surface patches. We state the equilibrium equations using the principle of virtual work and perform a consistent linearization without choosing of a particular constitutive equation. For example, we can then easily utilize the model proposed by Holzapfel et al. [2], one of the most popular constitutive model for soft collagenous tissues. The strain energy function of the Holzapfel material consists of an isotropic New-Hookean term for the matrix and an anisotropic contribution from a single family of fibers for a transversely isotropic material. We demonstrate the features of our formulation by means of several selected examples.

Our present formulation will allow scientists in the fields of computational biomechanics and mechanobiology to adopt arbitrary constitutive equations that have been originally developed for three-dimensional biological solids within the isogeometric shell framework. The numerous advantages of isogeometric analysis could allow us to design more efficient and robust computational tools for medical device design with applications to thin biological membranes.
REFERENCES
