## EIGENFREQUENCIES AND STABILITY OF ELASTIC AND VISCOELASTIC ACCELERATING PANELS WITH FLUID–STRUCTURE INTERACTION

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Models of axially moving materials, and especially of their out-of-plane vibrations, are commonly considered in the context of industrial production processes, such as paper making. Typical models include axially moving strings, beams, panels (plates with cylindrical deformation), membranes and plates. Research into the field began in 1897 [1]. Other important classical studies include e.g. [2, 3], and the field has remained active to this day; e.g. [4, 5]. Classically an elastic material model has been used, but in the recent years the focus of research has shifted to viscoelastic materials. Both linear and nonlinear models have been investigated in the literature.

Problems of out-of-plane behaviour of axially moving materials share some of their mathematical formulation with those of axially compressed stationary materials, leading to questions of stability. The problem parameter of interest is, typically, the axial velocity of the material, which is assumed constant.

However, it is known that in paper machines the axial tension of the paper web in open draws is generated by a velocity difference between adjacent rollers. Thus, an axial strain component is generated, and the material accelerates axially. Axially accelerating moving materials have been investigated since the 1970s, see e.g. [6]. Recent focus has been on accelerating viscoelastic materials; e.g. [7, 8].

In the case of lightweight materials, such as paper, one further consideration arises from the inertial contribution of the surrounding air. It is not sufficient to consider the vibrations of the moving material in vacuum, but the fluid–structure interaction must be accounted for. The surrounding air is known to change both the frequencies of natural vibration and the critical velocity [9]. In the present study, we investigate an axially accelerating panel subjected to an axial potential flow. The acceleration is considered to be purely space-dependent in the Eulerian reference frame, i.e. there is no time dependence. The flow component is handled via a Green's function solution [10], leading to a one-dimensional integrodifferential model. Both linear elastic and Kelvin–Voigt viscoelastic material models will be considered.

The eigenfrequency spectrum will be parametrically investigated as a function of the axial velocity, and the critical velocity will be determined. Numerical results with comparisons between the different cases will be presented.

## REFERENCES

- [1] Rudolf Skutch. Uber die Bewegung eines gespannten Fadens, weicher gezwungen ist durch zwei feste Punkte, mit einer constanten Geschwindigkeit zu gehen, und zwischen denselben in Transversal-schwingungen von gerlinger Amplitude versetzt wird. Annalen der Physik und Chemie, 61:190–195, 1897.
- [2] F. R. Archibald and A. G. Emslie. The vibration of a string having a uniform motion along its length. ASME Journal of Applied Mechanics, 25:347–348, 1958.
- [3] R. D. Swope and W. F. Ames. Vibrations of a moving threadline. Journal of the Franklin Institute, 275:36–55, 1963.
- [4] L. Kong and R. G. Parker. Approximate eigensolutions of axially moving beams with small flexural stiffness. *Journal of Sound and Vibration*, 276:459–469, 2004.
- [5] Y. Wang, L. Huang, and X. Liu. Eigenvalue and stability analysis for transverse vibrations of axially moving strings based on Hamiltonian dynamics. Acta Mechanica Sinica, 21:485–494, 2005.
- [6] C. D. Mote. Stability of systems transporting accelerating axially moving materials. ASME Journal of Dynamic Systems, Measurement, and Control, 97:96–98, 1975.
- [7] L.-Q. Chen, H. Chen, and C.W. Lim. Asymptotic analysis of axially accelerating viscoelastic strings. *International Journal of Engineering Science*, 46(10):976 985, 2008. DOI: 10.1016/j.ijengsci.2008.03.009.
- [8] Y.-F. Zhou and Z.-M. Wang. Dynamic stability of axially accelerating viscoelastic plate. Scholarly Research Exchange, 2009(2):1–6, 2009. DOI:10.3814/2009/856320.
- [9] A. Kulachenko, P. Gradin, and H. Koivurova. Modelling the dynamical behaviour of a paper web. Part II. Computers & Structures, 85:148–157, 2007.
- [10] N. Banichuk, J. Jeronen, P. Neittaanmäki, and T. Tuovinen. Dynamic behaviour of an axially moving plate undergoing small cylindrical deformation submerged in axially flowing ideal fluid. *Journal of Fluids and Structures*, 27(7):986–1005, 2011.