EFFICIENCY AND RELIABLE ERROR CONTROL FOR THE OBSTACLE PROBLEM

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Key words: Obstacle problem, adaptive algorithm, efficiency, reliability, optimality, Lagrange multiplier.

For the obstacle problem as the prototypical example for variational inequalities, the “Other Look” by Braess [1] at Lagrange multipliers has lead to a paradigm with reliable and partly efficient error control. Therein the efficiency is understood in terms of the error \( u - v \) in \( H^1_0(\Omega) \) for the exact solution \( u \) and an approximation \( v \) in \( H^1_0(\Omega) \) in the primal variable and the error \( \lambda - \mu \) for the exact Lagrange multiplier \( \lambda \) and an approximation \( \mu \) in the dual space \( H^{-1}(\Omega) \). These error terms are compared with explicit computable terms, as in the analysis for variational equalities. Reliability and efficiency then leads to the equivalence

\[
||| u - v ||| + ||| \lambda - \mu |||, \approx \text{computable terms}
\]

possibly up to multiplicative generic constants.

This talk presents a reliable and efficient a posteriori error analysis for the conforming finite element method (FEM) from [3]. The reliable error control is even a guaranteed upper bound for the exact error. The talk answers the question of efficiency beyond the aforementioned equivalence. Given the exact Lagrange multiplier \( \lambda \) for which choice of an approximation \( \mu \) of \( \lambda \) does it hold

\[
||| \lambda - \mu |||, \lesssim ||| u - v ||| + \text{data oscillations?}
\]

It clarifies the role of the Lagrange multiplier and possible choices for suitable approximations. Furthermore reliability and efficiency is viewed as the equivalence

\[
||| u - v ||| \approx \text{computable terms.}
\]

The results of the a posteriori analysis lead to an adaptive algorithm. The optimality for a conforming adaptive FEM is shown by Carstensen und Hu [2].
REFERENCES

