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EFFICIENCT AND RELIABLE ERROR CONTROL FOR THE OBSTACLE PROBLEM

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For the obstacle problem as the prototypical example for variational inequalities, the "Other Look" by Braess [1] at Lagrange multipliers has lead to a paradigm with reliable and partly efficient error control. Therein the efficiency is understood in terms of the error $u - v$ in $H^1_0(\Omega)$ for the exact solution u and an approximation v in $H^1_0(\Omega)$ in the primal variable and the error $\lambda - \mu$ for the exact Lagrange multiplier λ and an approximation μ in the dual space $H^{-1}(\Omega)$. These error terms are compared with explicit computable terms, as in the analysis for variational equalities. Reliability and efficiency then leads to the equivalence

$$|||u - v||| + |||\lambda - \mu|||_* \approx \text{computable terms}$$

possibly up to multiplicative generic constants.
This talk presents a reliable and efficient a posteriori error analysis for the conforming finite element method (FEM) from [3]. The reliable error control is even a guaranteed upper bound for the exact error. The talk answers the question of efficiency beyond the aforementionend equivalence. Given the exact Lagrange multiplier λ for which choice of an approximation μ of λ does it hold

$$|||\lambda - \mu|||_* \lesssim |||u - v||| + \text{data oscillationes?}$$

It clarifies the role of the Lagrange mulitplier and possible choices for suitable approximations. Furthermore reliability and efficiency is viewed as the equivalence

$$|||u - v||| \approx \text{computable terms.}$$

The results of the a posteriori analysis lead to an adaptive algorithm. The optimality for a conforming adaptive FEM is shown by Carstensen und Hu [2].

REFERENCES

[1] D. Braess. A posteriori error estimators for obstacle problems–another look. *Numer. Math.*, Vol. **3**, 2005

[2] C. Carstensen and J. Hu. Optimality of a conforming adaptive finite element method for an affine obstacle problem. (in preperation)

[3] C. Carstensen and K. Koehler. Discrete Lagrange multipliers in numerical simulations for the obstacle problem (work in progress)