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EIGENELEMENTS PARAMETRIC SENSITIVITY AND APPLICATION TO PROPER ORTHOGONAL DECOMPOSITION

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1 Abstract

Parametric reduced order models are very important for several applications including PDEs (Partial Differential Equations) of high dimensions ([Grepl et al., MMNA, 2007], [Nguyen et al., Calcolo, 2009], [Buffa et al., MMNA, 2011], [Chinesta et al., ACME, 2013], [Gonzalez et al., MCS, 2012]...). Indeed, one significant question concerning Proper Orthogonal Decomposition methods (POD) is : given a solution to a problem for a set of (physical) parameters, can we control the accuracy of the corresponding POD basis when the parameter set varies ? *A priori* estimates on the error induced by such techniques are performed in (Akkari et al., JCAM, 2013). In this communication, we propose a direct estimate of this error based on the control of the distance between the POD eigenspaces when the parameter is changing. More precisely, we show that while eigenvalues depend in a lipschitzian way in self-adjoint compact operators, the eigenspaces are only locally Lipschitz.
Therefore, given X a Hilbert space, and $\mathcal{U}_\lambda = \{u_\lambda(t), t \in [0,T]\} \subset L^2(0,T;X)$, where $\lambda \in \mathbb{R}^p$, $p \in \mathbb{N}^*$, denotes a parameter of this family. Let $\left(\Phi_n^\lambda\right)_{n\geq 1}$ be a POD basis associated with $\mathcal{U}_\lambda$, then Φ_n^λ is a unit vector which is the solution of the following eigenvalue problem :

$$\mathcal{R}(u_\lambda)\,\Phi_n^\lambda = \mu_n^\lambda\,\Phi_n^\lambda, \text{ with } \mathcal{R}(u_\lambda)\,\varphi = \frac{1}{T}\int_0^T (u_\lambda(t),\varphi)_X\, u_\lambda(t)dt.$$

Let $E(\mu_n^\lambda)$ be the eigensubspace associated with the POD eigenvalue μ_n^λ, and $\left\|E(\mu_n^\lambda) - E(\mu_n^{\lambda_0})\right\|$

a certain *gap* between the two subspaces $E(\mu_n^\lambda)$ and $E(\mu_n^{\lambda_0})$.

If the family $\mathcal{U}_\lambda$, verfies the following :

$$\|u_\lambda - u_{\lambda_0}\|_{L^2(0,T;\Omega)} \leq K \|\lambda - \lambda_0\|^\alpha , \tag{1}$$

a key question is : What can be said about $\left\| E(\mu_n^\lambda) - E(\mu_n^{\lambda_0}) \right\|$?

In our presentation, we will

– give an answer to this question after giving a generalization of the main result obtained by B. Rousselet and D. Chenais [Rousselet et al., AMO, 1990].

– apply this result for a class of parametrized quasi-linear parabolic equations whose solutions verify the condition (1).

– show how such control of the distance between the POD eigensubspaces is used to estimate the error relatively to the models reduction by a reference POD basis when parametric evolution occurs.