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Toward cyclic plasticity with X-FEM : a new integration method avoiding field projection in the elements cut by a crack.

Jean-Baptiste Esnault[1,2], Alexandre Martin[2*] and Patrick Massin[2]

[1] LMS, UMR 7649, Ecole Polytechnique, Palaiseau France, esnault@lms.polytechnique.fr.
[2] LaMSID, UMR EDF-CNRS-CEA 8193, 1 Avenue du Général de Gaulle, 92141 Clamart Cedex, alexandre-externe.martin@edf.fr, patrick.massin@edf.fr.

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The introduction of plasticity in an X-FEM model is very recent [1, 2] and arises several technical and scientific difficulties. Currently, the existing analyses in plasticity with X-FEM is limited to monotonic plasticity which is approximated by non-linear elasticity using the HRR field decomposition [2]. Such an analysis cannot be applied in fatigue where the cyclic plasticity plays a major role in crack propagation. The aim of this study is to tackle one of the difficulties induced by a cyclic plastic computation: the projection of the internal variables during crack propagation.

Indeed, the classical Gaussian quadrature method, widely used to evaluate numerically the value of the stiffness matrix and the internal forces, implies a polynomial approximation of the integrand (stress, strain). In the framework of Finite Element method expressions of shape functions and their derivatives at Gauss points are used. And yet in the X-FEM methodology, in order to take into account the singularities induced in terms of displacement in the elements cut by a crack, the interpolation basis is enriched with discontinuous functions [3]. Computing directly on the whole elements the value of the stiffness matrix and the internal forces is thus impossible, the elements cut by the crack being partitioned into integration sub-cells on each side of the discontinuity where the Gaussian quadrature is applied.

This subdivision of the parent element into integration sub-cells is not well suited to treat plasticity for two main reasons:
- It implies field projections of the internal variables (which take into account the loading history) from the parent element to the integration sub-cells which is associated to the difficult task of finding a conservative projection method in terms of energy.
- It increases the number of integration points necessary to compute the elementary stiffness matrix which increases the computational cost.

For those reasons, we try in this study to eliminate the quadrature sub-cells in the elements cut by the crack introducing a new integration method based on a previous work by Ventura in [4]. We thus focus in a first place on interface problems.

The discontinuous Heaviside function in the elements cut by the crack is replaced by a new continuous enrichment function. This new function is polynomial and its degree and coefficients depend on the type of element and on the position of the crack in the parent element. The automatic determination of the new enrichment and its introduction in a linear and non-linear framework has been implemented for linear elements in two and three dimensions.

The potentiality of this new integration method is demonstrated with convergence tests in elasticity: despite a significant drop in the number of integration points, the convergence rates are closed to the theoretical values. A convergence test in plasticity with an elastic-perfectly plastic hollow sphere submitted to uniform internal pressure, has been implemented and the efficiency of the new scheme is currently tested on this test-case.

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