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A NUMERICALLY STABLE A POSTERIORI ERROR ESTIMATOR FOR REDUCED BASIS APPROXIMATIONS OF ELLIPTIC EQUATIONS

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Many problems in science and engineering require the solution of partial differential equations on large computational domains or very fine meshes. Even on modern hardware, standard discretization techniques for solving these problems can require many hours or even days of computation, which makes these approaches inapplicable for many-query situations like, e.g., design optimization, where the same problem has to be solved many times for different sets of parameters.

The Reduced Basis Method (RB) is by now a well-established tool for the model order reduction of problems formulated as parameterized partial differential equations (see e.g. [1, 2]). In an "offline phase", a given high-dimensional discretization is solved for appropriately selected parameters and a reduced subspace is constructed as the span of these solution snapshots. In a later "online phase", the problem can be solved efficiently for arbitrary new parameters via Galerkin projection onto the precomputed reduced space.

One crucial ingredient for the application of RB schemes is the availability of an a posteriori error estimator to reliably estimate the error introduced by the reduction process. Such an estimator is also required by the weak greedy algorithm, which has been shown to be optimal for the generation of the reduced spaces [3], to efficiently perform an exhaustive search of the parameter space for parameters maximising the reduction error.

For affinely decomposed elliptic problems, a residual based error estimator is widely used [1, sec. 4.3]. However, as observed by several authors [1, pp. 148–149][4][5], the implementation of this estimator shows poor numerical accuracy due to round-off errors which can render the estimator unusable when the given problem is badly conditioned and small reduction errors are required. Using calculations with higher-precision floating point numbers has poor computational performance. An alternative algorithm to evaluate the estimator, which is numerically stable, has been proposed in [6, 7], which however also comes at the price of either a more expensive "online phase" and/or increased complexity of offline computations.

In this talk we propose a new algorithm based on representing the residual w.r.t. a dedicated orthonormal basis, which is both easy to implement and comes with no additional online cost as well as small additional offline overhead. We will perform a numerical analysis of our algorithm comparing it with the standard one and present some numerical examples to demonstrate its performance. Moreover, we will indicate that choosing a smaller basis for representing the residual might even lead to faster estimator evaluations compared to the standard algorithm without sacrificing its high accuracy.

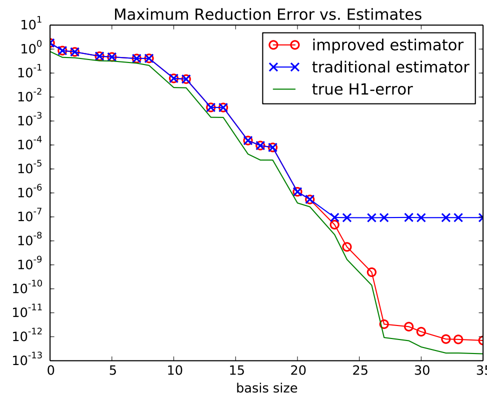

Figure 1: Maximum error vs. estimated errors for an elliptic "2x2-Thermalblock Problem" with parameter space $[0.1, 1]^4$.

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