

A HIGH-ORDER FINITE ELEMENT APPROACH FOR TREATING MULTI-MATERIAL ZONES IN ALE HYDRODYNAMICS

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In multi-material arbitrary Lagrangian-Eulerian (ALE) simulations, where complex material interfaces are present, it is oftentimes necessary to employ so-called *mixed zones* where a single computational element contains multiple materials. Such zones can be generated during the re-meshing phase of ALE or even at the start of a simulation when aligning the mesh to the material interfaces is not possible or desirable.

The common way of treating mixed zones consists of duplicating the material state data, such as density, energy, etc., for every material present in a zone. In addition, a set of new variables, e.g. volume fractions, is added to the zonal state data that describes the distribution of the materials within the zone. In this process however, only a single material velocity field is used, motivated by the fact that the conforming computational mesh has a single motion during the Lagrangian phase of ALE. The goal of a multi-material algorithm is then to define the evolution of all state data during the Lagrangian and remap phases of ALE.

There are many existing approaches to treating multi-material zones, see for example [1, 2] and the references therein. In this talk, we present extensions and new developments for mixed zone algorithms for the case of high-order finite element discretizations of the hydrodynamic equations described in [3]. To this end, we re-formulate existing methods for the low-order, cell-centered case (e.g. [1]) as a multi-material mathematical models on a continuous level, thus extracting and separating the model from the discretization algorithm. We then proceed to apply a high-order finite element discretization to this continuous model.

In the typical case where volume fractions are used to describe the material distribution in a mixed zone, we consider, at a continuous level, a set of material indicator (or volume fraction) functions that are non-negative and sum to one. Using the material specific density, specific internal energy and indicator functions, combined with the common velocity field, we formulate the conservation equations of mass and energy for each material plus a single equation for momentum conservation. To complete the system, we add an evolution equation for the material indicators based on classical closure models.

In this talk, we will present the high-order spatial and temporal discretization of the multi-material equations and illustrate the numerical performance of the resulting algorithms on a set of model test problems.

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