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AN ENERGY CONSISTENT APPROACH FOR ELASTODYNAMIC FRICTIONAL CONTACT PROBLEMS

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Two important topics to solve dynamic frictional contact problems concern the enforcement of suitable contact interface conditions and the obtention of energy consistent properties. These two issues lead both to propose acceptable physical models and to elaborate numerical schemes with long term time integration accuracy and stability properties. During the last twenty years, many works [1, 8, 4, 6, 7, 2, 5] have been devoted to the construction of energy conserving methods for elastodynamic contact problems and sometimes with consistent energy dissipation extensions to the case of friction [8, 7, 2].
The choice of suitable physical contact conditions are not so trivial as it for the contact and impact problems. In this work, we propose to establish frictional contact conditions which respect both to some microscopic properties of the contact surface (as asperities) and to some energy consistent properties. For these reasons, we consider a specific contact model based on normal compliance and unilateral constraint which provide intrinsic energy consistent properties characterized by a conserving behaviour for frictionless impacts and admissible dissipation for friction phenomenon. The contact conditions is modeled with a normal compliance condition of such a type that the penetration is limited with unilateral constraint and is characterized by the following relations:

$$\left.\begin{array}{l} u_\nu \le g, \quad \sigma_\nu + p(u_\nu) \le 0, \\ (u_\nu - g)(\sigma_\nu + p(u_\nu)) = 0 \end{array}\right\} \quad \text{on} \quad \Gamma_3 \qquad (1)$$

Condition (1) represents an unilateral contact with finite penetration in which the compliance function p can take the following form: $p(r) = cr_+$, where $p(\cdot)$ is a nonnegative prescribed function which vanishes for negative argument and c is a positive constant related to the stiffness of a surface contact foundation characterized by a thin layer of asperities of thickness g related to the penetration.
The model of friction which represents a generalization of the model proposed in [3], is based on a specific Coulomb law adapted to the contact conditions (1) with normal compliance and unilateral constraint. In this case, we assume that the coefficient of friction can depend both on the depth of the penetration u_ν and on the tangential slip rate $\dot{\boldsymbol{u}}_\tau$. Furthermore, the friction bound depends also on the effective normal contact stress; note

that the normal contact stress σ_ν is equal to the normal compliance stress $-p(u_\nu))$ until the penetration u_ν reaches the limit g.

$$\left.\begin{aligned} &\|\boldsymbol{\sigma}_\tau\| \leq -\mu(u_\nu, \|\dot{\boldsymbol{u}}_\tau\|)\, \sigma_\nu, \\ &-\boldsymbol{\sigma}_\tau = -\mu(u_\nu, \|\dot{\boldsymbol{u}}_\tau\|)\, \sigma_\nu \frac{\dot{\boldsymbol{u}}_\tau}{\|\dot{\boldsymbol{u}}_\tau\|} \;\; \text{if} \;\; \dot{\boldsymbol{u}}_\tau \neq \mathbf{0} \end{aligned}\right\} \quad \text{on} \quad \Gamma_3 \tag{2}$$

In order to provide energy consistent properties related to the specific frictional contact given in (1) and (2), the next step is to focus on computational aspects with accuracy and stability properties. Then, we propose an energy-consistent scheme based on recent energy-controlling time integration methods for nonlinear elastodynamics [1, 8, 4, 6, 7, 2, 5]. In particular, a newton continuation method [2] with an augmented Lagrangian technic is used for the enforcement of the contact condition (1). Furthermore to obtain additional energy conserving properties, we combine the specific penalized methods proposed in [6, 2] during the flattening of the asperities with the procedure of equivalent mass matrix [4, 5]. Some numerical experiments of representative impact problems are considered to characterize the good properties both of the contact model and the numerical scheme in terms of conservation of energy. In particular, we study the behaviour of our numerical modelization compared to some existing time integration schemes [4, 6, 2].

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