

# A RESIDUAL-BASED VARIATIONAL MULTISCALE DISCONTINUOUS GALERKIN METHOD FOR TURBULENT FLOWS

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Turbulent flows in aeronautical applications typically have a very wide range of length and time scales characterized by their high Reynolds numbers. Traditionally, the prohibitively large computational cost implied by a high Reynolds number has made physical *modelling* rather than direct *computation* of turbulent scales a more popular approach in the industry. While well-calibrated turbulence models can provide good accuracy at reasonable computational costs, they tend to have a very limited range of applicability. In particular, flows with inhomogeneous features sensitive to small errors, such as flow with detachment, cannot be accurately predicted by conventional models and require computing a greater range of turbulent scales.

*Large-eddy simulation* (LES) aims to resolve all relevant turbulent scales while employing more fundamental models to account for the influence of the unresolved scales. Therefore, it has great potential for a more *universal* turbulence simulation methodology that is valid for any configuration. In order to make LES a tool of engineering practice, it is necessary to address issues of computational cost, handling of complex geometries and compatibility of the employed models with the used numerical method. In this work, we propose a residual-based variational multiscale discontinuous Galerkin method to address these issues and to ultimately improve efficiency by exploiting its flexibility in local refinement.

The *variational multiscale* (VMS) [1] approach is a framework for designing numerical methods and associated models for multiscale phenomena. In this approach, the scales not resolved by the numerical scheme are modelled by a fine scale model. This approach is particularly well-suited for higher order methods, since the model term does not add any low order consistency errors. In the context of LES, many VMS variants have been proposed, with *implicit* [2] and *explicit* [3] fine scale models. The implicit models have the

disadvantage of introducing additional degrees of freedom for the fine scales, leading to a three-level decomposition of scales. In this work we concentrate on extending the cheaper explicit residual-based fine scale model developed in the framework of continuous finite element methods [3] to discontinuous Galerkin (DG) [4] methods to benefit from their excellent scalability properties in simulating unsteady phenomena on parallel computers. DG methods have been successfully used in the variational multiscale setting for the large-eddy simulation of turbulence [5, 6] with implicit fine scale models. In this work we propose a cheaper explicit fine scale model based on a scaling of the cell and face residuals of the resolved scales.

We numerically explore various options for constructing residual-based fine scale models for DG discretizations. These methods differ in how (or whether) the jump residuals are taken into account for the fine scale model. We compare these options using the viscous Burgers' equation in 1D to note that residual-based VMS DG with even the simplest fine scale model that disregards interface jumps gives results that are better than the plain DG method for underresolved simulations.

In the second part of this study, we apply the proposed methodology to the compressible Navier-Stokes equations. We perform LES of wall-bounded turbulence based on the proposed residual-based VMS DG method and compare our results to the results obtained through implicit LES and LES based on the three-level VMS DG method in terms of accuracy and computational cost. We conclude that the proposed residual-based VMS DG method is competitive with the three-level VMS DG in terms of accuracy while retaining only the coarse scale degrees of freedom. In terms of computational cost, the proposed method is better than the three-level VMS and competitive with implicit LES.

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