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HYBRIDIZED DISCONTINUOUS GALERKIN METHODS FOR LARGE EDDY SIMULATION OF TURBULENT FLOW

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Discontinuous Galerkin (DG) methods have several properties that make them attractive for the simulation of fluid flow: They naturally embed the physical directionality of transport by a balance of fluxes into and out of cells in a high order framework. Hence, DG methods work well also for the convection-dominated problems ubiquitous in fluid dynamics, as opposed to continuous finite element methods that require (non-trivial) stabilization. However, DG schemes typically involve considerably more coupled degrees of freedom for the same level of resolution, as compared to finite elements or finite volumes.

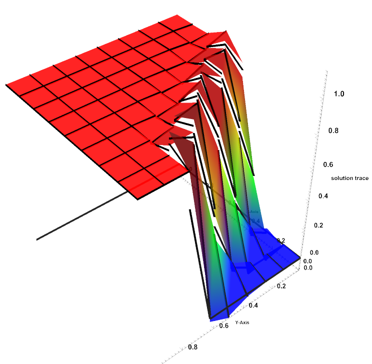

Figure 1: Representation of discontinuity by interior DG solution (surfaces) and trace solution (black lines) in 2D.

Hybridized DG (HDG) methods try to mitigate the cost disadvantage of DG in implicit settings by reducing the final linear system to degrees of freedom on element faces [3]. These trace variables are formally introduced to describe the numerical flux over the element boundaries. The usual DG variables in the element interiors are eliminated during element assembly by static condensation. In the HDG method for the incompressible Navier–Stokes equations, standard discontinuous polynomial spaces of tensor degree k for the velocity $\mathbf{u}$, velocity gradient $\mathbf{L}$ and pressure p are chosen. For the trace velocities, tensor product polynomials of degree k are chosen on each face of the triangulation. For the pressure, only the average pressure per element couples globally. This gives a linear system of the size $|(Q_k^{d-1})^d| \times \text{n\_faces} + \text{n\_elements}$ degrees of freedom for the trace velocity and average pressure. With appropriate definition of fluxes, this combination of standard spaces yields optimal convergence rates $k+1$ in all three solution variables $\mathbf{u}, \mathbf{L}, p$ (see [3]), and rate $k+2$ for a post-processed velocity.

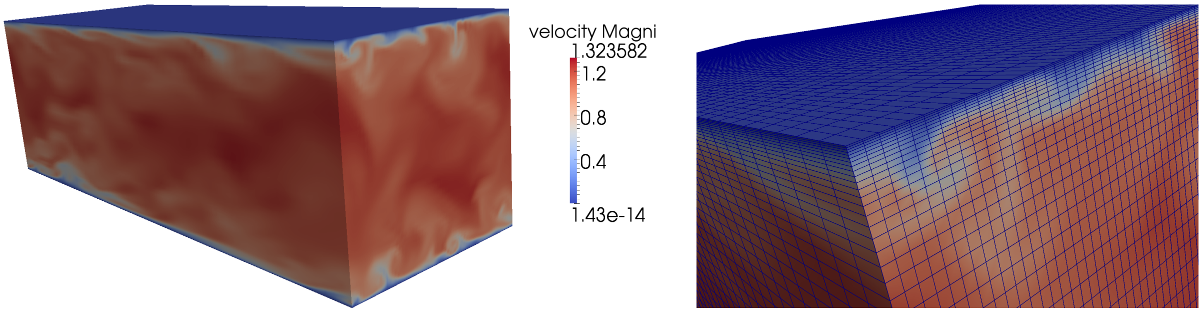

Figure 2: LES of turbulent channel flow with mesh adapted to the wall boundary layer.

In this work, we extend the HDG method to the large eddy simulation (LES) of three-dimensional turbulent channel flow. Our LES approach uses a small-scale subgrid viscosity term inspired by the term derived using the variational multiscale method in [1]. For the HDG method, small-scale strains for the subgrid viscosity can easily be extracted from the local discontinuous velocity gradient variable $\mathbf{L}$ by polynomial scale separation. We compare the HDG results to the ones obtained with stabilized finite elements and the LES model of [1]. In particular, the quality of velocity data (mean, rms values) is compared for the original DG variables and the post-processed quantities. For linear polynomials, $k = 1$, we also show results of an embedded DG method instead of HDG that enforces continuity of trace variables over face boundaries [4]. Such a method has the same number of velocity degrees of freedom in the final linear system as continuous finite elements.

Despite the embarassingly parallel character of the element computations and element-by-element elimination of local variables, we found that a naive implementation of assembly using quadratic and cubic polynomials typically requires an order of magnitude more computational time than actually solving the linear system with an efficient solver on hexahedra. This necessitates the use of fast assembly techniques as discussed in [2] for the implicit solution of the Navier–Stokes equations where the system matrix changes in every nonlinear iteration step.

REFERENCES

[1] V. Gravemeier, M. W. Gee, M. Kronbichler, W. A. Wall. An algebraic variational multiscale–multigrid method for large eddy simulation of turbulent flow. *Comput. Meth. Appl. Mech. Engrg.*, Vol. **199**, 853–864, 2010.

[2] M. Kronbichler, K. Kormann. A generic interface for parallel cell-based finite element operator application. *Comput. Fluids*, Vol. **63**, 134–147, 2012.

[3] N. C. Nguyen, J. Peraire, B. Cockburn. An implicit high-order hybridizable discontinuous Galerkin method for the incompressible Navier–Stokes equations. *J. Comput. Phys.*, Vol. **230**, 1147–1170, 2011.

[4] J. Peraire, N. C. Nguyen, B. Cockburn. An embedded discontinuous Galerkin method for the compressible Euler and Navier–Stokes equations. *20th AIAA Computational Fluid Dynamics Conference* AIAA 2011-3228, 2011.