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CURVED HEXAHEDRAL MESH GENERATION BY SMOOTHING ACCORDING TO THE CAD HIERARCHY

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During the last years, unstructured high-order methods are gaining popularity in those applications of computational methods where high-fidelity simulations on curved domains are required. For these applications, it is required to generate meshes composed by high-order elements that are curved to match the boundaries of the domain. The existent techniques to generate curved meshes are based on *a posteriori approach*. Specifically, an unstructured linear mesh is first generated. Then, the linear mesh is converted to a high-order mesh, and curved to match the domain boundary. Finally, the boundary nodes are fixed, to preserve the curved boundary mesh, and the inner nodes are relocated to obtain a valid and high-quality mesh.

The standard implementation of the *a posteriori* approach starts from an unstructured tetrahedral mesh, since their generation is more robust and automatic than the generation of hexahedral elements. Nevertheless, hexahedra present several practical advantages that emerge from its natural tensorial structure. First, they can be aligned to accurately approximate the anisotropic features of a solution field, such as the boundary layer present on the viscous regions of a flow field. Second, shape basis functions are obtained as the tensor product of one-dimensional basis functions, such as polynomials (high-order methods), B-splines, and NURBS (iso-geometric analysis). Third, quadrature rules are obtained as the tensor product of one-dimensional quadrature rules. Fourth, the computational cost of computing elemental matrices can be reduced by using techniques as the sum factorization method [1]. To exploit these advantages, it is required to generate curved hexahedral meshes. However, few works have addressed this issue.

The main limitation of the standard *a posteriori* approach, is that the quality of the final curved mesh is constrained by the quality of the curved mesh on the boundary obtained in the second step. That is, in the final step, the interior nodes are moved to improve the quality of the curved elements. However, the boundary nodes obtained in the second step are fixed and therefore, the final quality is limited by this constraint. To enhance the quality, it is required to relocate both the nodes in the interior and the boundary of the mesh taking into account the curved edges and surfaces of the CAD model.

To address these issues, we propose an *a posteriori* approach to generate curved and high-order hexahedral meshes. Specifically, we propose to generate the initial linear hexahedral mesh with well-known algorithms, such as the submapping [2] or the multi-sweeping [3] methods. Second, we curve the mesh boundary to match the domain boundary. Third, we optimize the quality of the whole mesh by moving both the interior and boundary nodes. To this end, we optimize a regularized distortion measure of the mesh expressed in terms of the coordinates of the nodes on the edges and surfaces of the CAD model, as well as the interior nodes. The meshes obtained with this approach are composed by valid curved and high-order hexahedral elements that approximate the curves and the surfaces of the domain boundary. Moreover, since the interior and boundary nodes are relocated, the final mesh quality improves the quality obtained with a standard *a posteriori* curved mesh generation approach.

We present several examples to illustrate the application and advantages of the method. Note that the resulting meshes are of practical interest for general unstructured high-order methods such as continuous and discontinuous Galerkin methods. However, their main application are those unstructured high-order methods that exploit the tensor structure of hexahedral elements such as the spectral element method and the iso-geometric analysis.

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