11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

APPLICATIONS OF EFFICIENT PARALLEL K-EXACT FINITE VOLUME RECONSTRUCTION ON UNSTRUCTURED GRIDS

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Key words: *Finite Volume Method, k-Exact Reconstruction, Parallel Computing, Unstructured Grids*

Over 20 years ago, Barth and Frederickson [1] presented a high order algorithm for Euler's equations on arbitrary unstructured meshes. This finite volume extension of Godunov's method breaks down naturally into three operations: high order reconstruction of the solution in each cell, high order numerical integration of the flux through each face, and high order time integration. High order reconstruction is achieved by the requirement of k-exactness: if the solution u is a polynomial of degree k on a certain neighborhood of a cell, then the reconstruction in that cell must coincide with u.

For $k = 1$, this approach gives classical 2^{nd} order schemes widely used in industrial and commercial software packages such as Cedre and Flusepa [1]. However, 2^{nd} order accuracy becomes increasingly insufficient with the growing use of LES modeling. However, the simple method [1] has a drawback for $k > 1$: To reduce computational complexity, the reconstruction on a grid cell should make use of data in adjacent cells only. On the other hand, high order reconstruction requires sufficiently many data samples, which means that data from cells beyond adjacent cells in the grid must be accessed. Many numerical schemes avoid this by supplementing or replacing the cell averages with new degrees of freedom allowing the algorithm to work with local data only, as is the case in Discontinuous Galerkin, Spectral Finite Volume and Residual Distribution schemes. Nevertheless, the simplicity of k-exact reconstruction in its classical form [1] makes this method very attractive for industrial applications. What is lacking is a way to compute a high order k-exact reconstruction from non local data in an efficient way.

[1] Cedre, developed at Onera, and Flusepa, developed at Astrium, are multiphysics packages for the compressible Navier Stokes equations on general unstructured grids in 2D and 3D.

Consider a domain $\Omega \subset \mathbb{R}^d$ with a mesh of N general polyhedra and note $\overline{u}_\alpha$ the average of a function u over cells numbered $1 \leq \alpha \leq N$. A k-exact reconstruction of u in cell α from a neighborhood $\mathbb{W}_\alpha = \{\beta_1, \ldots, \beta_m\}$ with $m = \sharp\mathbb{W}_\alpha$ and $\alpha \in \mathbb{W}_\alpha$, is given by k linear applications $\boldsymbol{w}_\alpha^{(k|l)}$ with coefficients $w_{\alpha\beta}^{(k|l)}$ such that the following holds whenever u is a polynomial of degree k on $\mathbb{W}_\alpha$:

$$\boldsymbol{w}_\alpha^{(k|l)} \left(\overline{u}_{\beta_1}, \ldots, \overline{u}_{\beta_m}\right) = \sum_{\beta \in \mathbb{W}_\alpha} w_{\alpha\beta}^{(k|l)} \overline{u}_\beta = D^{(l)} u\Big|_{\boldsymbol{x}_\alpha} \quad , \; 1 \leq l \leq k \; . \tag{1}$$

Note that (1) is impossible if $m = \sharp\mathbb{W}_\alpha < \binom{k+d}{d}$. To avoid this restriction, we make the following observation: given k-exact $\boldsymbol{w}_\beta^{(k|k)}$ for $\beta \in \mathbb{W}_\alpha$ with small $m = \sharp\mathbb{W}_\alpha$, there are coefficients $\mathfrak{D}_{\alpha\beta}^{(k+1)}$ such that whenever u is a polynomial of degree $k+1$:

$$\boldsymbol{w}_\alpha^{(k+1|k+1)} \left(\ldots, \overline{u}_{\beta_i}, \ldots\right) = \sum_{\beta \in \mathbb{W}_\alpha} \mathfrak{D}_{\alpha\beta}^{(k+1)} \boldsymbol{w}_\beta^{(k|k)} \left(\ldots, \overline{u}_{\gamma_j}, \ldots\right) = D^{(k+1)} u\Big|_{\boldsymbol{x}_\alpha} \; . \tag{2}$$

Starting with given $\boldsymbol{w}_\beta^{(1|1)}$, the algorithm computes first $\boldsymbol{w}_\beta^{(2|2)}$, then $\boldsymbol{w}_\beta^{(3|3)}$ and so on using only sums over small neighborhoods in (2). *This eliminates the need to handle large neighborhoods which simplifies the implementation on parallel computers.* It can be shown that the $\mathfrak{D}_{\alpha\beta}^{(k+1)}$ exist on uniform cartesian meshes. On tetrahedral and general polyhedral meshes, numerical computations have been necessary to confirm the existence of the $\mathfrak{D}_{\alpha\beta}^{(k+1)}$.

In 2012, results with CEDRE have been presented for the simple test case of the advection of an inviscid vortex in 2D [2]. Since then, the implementation of high order limiters and positivity preserving mechanisms [3] in CEDRE have made it possible to compute more demanding applications up to 4^{th} order. In particular, we will present the simulation of a jet on a tetrahedral grid and the computation of multi-species flow in a supersonic mixing layer on a cartesian grid in 2D.

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